**Topic: Circular Measure**

**Measuring angles in radians**

“A radian is the angle subtended at the centre of a circle by an arc length whose length is equal to that of the radius of the circle”. This means a radian is the angle formed when the arc length and the radius are the same.

\[
\text{No. of Radians in a circle} = \frac{\text{length of circumference}}{\text{radius}} = \frac{2\pi r}{r} = 2\pi
\]

\[
\therefore 360^\circ = 2\pi \text{ rads} \quad 180^\circ = \pi \text{ rads} \quad 1\text{ rad} = \frac{180}{\pi} \approx 57.3^\circ
\]

**Converting angles from Degrees to Radians**

1. Convert 45° to radians

\[
45^\circ \times \frac{\pi}{180} = \frac{45\pi}{180} = \frac{\pi}{4}
\]

Leaving your answer in terms of \(\pi\) (exact)

2. Convert 75° to radians

\[
75^\circ \times \frac{\pi}{180} = \frac{75\pi}{180} = 1.308... \approx 1.31
\]

Leaving your answer in 3 sig. fig. (approximated)

**Converting angles from Radians to Degrees**

1. Convert \(\frac{2\pi}{3}\) rads to degrees

\[
\frac{2\pi}{3} \times \frac{180}{\pi} = 120^\circ
\]

2. Convert 2.1 rads to degrees

\[
2.1 \times \frac{180}{\pi} = 120.3^\circ
\]
Finding Arc Length

When angles are measured in degrees:

Length of Arc $XY$, $s = \frac{\theta}{360} \times 2\pi r$

When angles are measured in radians:

Length of Arc $XY$, $s = \frac{\theta}{2\pi} \times 2\pi r = r\theta$

1. Find the length of the arc $AB$.

   
   \[
   \text{Length of Arc } AB = r\theta \\
   = 5\left(\frac{2\pi}{3}\right) \\
   = \frac{10\pi}{3} \text{ cm}
   \]

2. Find the radius of the sector $ABC$.

   \[
   \text{Length of Arc } AB = r\theta \\
   16 = r\left(\frac{5\pi}{4}\right) \\
   r = \frac{64}{5\pi} \text{ cm}
   \]

3. An arc $AB$ of a circle, centre $O$ and radius $r$, subtends an angle of $\theta$ radians. Given that the perimeter of the sector $AOB$ is $P$ cm, express $r$ in terms of $P$ and $\theta$.

   \[
   P = 2r + r\theta \\
   P = r(2 + \theta) \\
   r = \frac{P}{2 + \theta} \text{ cm}
   \]
Finding Area of Sector

When angles are measured in degrees:

Area of Sector $OXY, A = \frac{\theta^\circ}{360} \times \pi r^2$

When angles are measured in radians:

Area of Sector $OXY, A = \frac{\theta}{2\pi} \times \pi r^2$

$$= \frac{1}{2} r^2 \theta$$

1. Find the area of the sector $ABC$, where $\angle ABC = \frac{\pi}{3}$ and $r = 2$ cm.

$$\text{Area of Sector } ABC = \frac{1}{2} (2)^2 \left( \frac{\pi}{3} \right)$$

$$= \frac{2\pi}{3} \text{ cm}^2$$

2. Find the area of the sector $ABC$, where $\angle ABC = 60^\circ$ and $r = 8$ cm.

Remember to convert the angle to radians first!

$$\text{Area of Sector } ABC = \frac{1}{2} (8)^2 \left( \frac{\pi}{3} \right)$$

$$= \frac{32\pi}{3} \text{ cm}^2$$

Finding the Area of Segment

Area of segment = Area of sector – area of a triangle

Area of Segment $$= \frac{1}{2} r^2 \theta - \frac{1}{2} r^2 \sin \theta$$

$$= \frac{1}{2} r^2 (\theta - \sin \theta)$$
1. Find the area of the shaded segment.

\[
\text{Area of segment} = \frac{1}{2}r^2(\theta - \sin \theta)
\]
\[
= \frac{1}{2}(9)^2\left(\frac{\pi}{6} - \sin \frac{\pi}{6}\right)
\]
\[
= 0.95575...
\]
\[
\approx 0.956 \text{ cm}^2
\]

Exercise

For all questions, give all answers for \( \theta \) in radians.

Q1 The diagram shows part of a circle, centre \( O \) and radius 7 cm. \( \angle POQ \) is 1.2 radians. Find the area and perimeter of the shaded region.

\[O \quad \theta
\]
\[Q \quad P \quad O \]
\[7 \quad 1.2 \quad 9
\]

Q2 In a sketch of a parachute below, \( APB \) is an arc centre \( X \), radius 12 cm and \( \angle AXB = 0.8 \) radians. \( AQB \) is an arc centre \( Y \), radius 6 cm and \( \angle AYB = \theta \) radians. Calculate the
(i) value of \( \theta \),
(ii) area of the shaded region.

\[Q \quad A \quad B \]
\[\theta \quad \theta
\]
\[O \quad 6 \quad 12 \quad 0.8
\]

Q3 In the diagram, the sector \( OAB \) has centre \( O \), radius 9 cm and \( \angle AOB = \frac{\pi}{6} \) radians. \( OC \) bisects \( \angle AOB \) and \( M \) is the midpoint of \( OC \). An arc \( PQ \) with centre \( M \) is drawn.

(a) Find \( \angle OMP \).

(b) Calculate the perimeter of the shaded region.
Q4 The diagram below shows a semi-circular school field, centre O. During National Day, the uniform groups assemble in the shaded regions of the field. Given that the radius is 30 m, arc $AB = arc\ CD = arc\ EF = 15$ m, find $\angle BOE$ and hence, the perimeter of the shaded region.

Q5 The figure below shows two circles, centres $X$ and $Y$, radii $r_1$ cm and $r_2$ cm which touches externally at $P$. Given that $\angle AXP = \frac{\pi}{4}$ radians, $\angle BYP = \frac{\pi}{3}$ radians and $AB$ is parallel to $XPY$,

(a)  (i) Express $h$ in terms of $r_1$.
     (ii) Express $h$ in terms of $r_2$.
     (iii) Hence, show that $\frac{r_1}{r_2} = \frac{1}{2\sqrt{6}}$.

(b) Given further that $r_1 = 6$,

     (i) find the perimeter of the shaded region
     (ii) find the area of the shaded region.

Q6 The diagram shows a sector $OPQRS$ with centre $O$. Arcs $PQ$, $QR$ and $RS$ have the same length. Given that $OP = 2$ m, $MN = 0.728$ m and

$\angle POS = \frac{2\pi}{3}$,  

(a) show that $OM = 1.064$ m,
(b) find the perimeter of the shaded region,
(c) find the area of the shaded region.