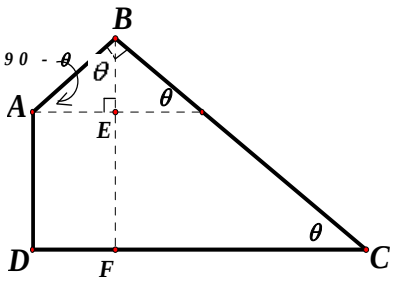
Year 4 Assignment 5 Solutions

Q1



$$\angle ABE = \theta$$

$$\sin\theta = \frac{AE}{AB} = \frac{AE}{1} \Rightarrow AE = \sin\theta$$

$$\cos\theta = \frac{BE}{AB} = \frac{BE}{1} \Rightarrow BE = \cos\theta$$

(i) $\text{Perimeter} = AB + BC + CD + DA$

$$= 1 + 7 + (CF + FD) + (BF - EF)$$

$$= 1 + 7 + (CF + AE) + (BF - BE)$$

$$= 1 + 7 + (7\cos\theta + \sin\theta) + (7\sin\theta - \cos\theta)$$

$$= 8 + 6\cos\theta + 8\sin\theta$$

(ii) $\text{Perimeter} = 8 + 10\cos\left[\theta - \tan^{-1}\left(\frac{8}{6}\right)\right]$

The maximum value of the perimeter is 18 cm

Q2

$$3\cos 2x + \sqrt{3}\sin 2x = -1$$

$$2\sqrt{3}\cos\left(2x - \tan^{-1}\frac{\sqrt{3}}{3}\right) = -1$$

$$\cos\left(2x - \tan^{-1}\frac{\sqrt{3}}{3}\right) = -\frac{1}{2\sqrt{3}}$$

$$\alpha = 73.2213...$$

$$30° < 2x - \tan^{-1}\frac{\sqrt{3}}{3} < 330°$$

$$2x - \tan^{-1}\frac{\sqrt{3}}{3} = 180° - \alpha, 180° + \alpha$$

$$x = 68.4°, 141.6°$$

Q3

$$3\sin\theta + 4\cos(60° - \theta) = 2$$

$$3\sin\theta + 4\left[\cos 60°\cos\theta + \sin 60°\sin\theta)\right] = 2$$

$$3\sin\theta + 2\cos\theta + 2\sqrt{3}\sin\theta = 2$$

$$\left(2\sqrt{3} + 3\right)\sin\theta + 2\cos\theta = 2$$

$$\sqrt{25 + 12\sqrt{3}}\sin\left(\theta + \tan^{-1}\frac{2}{2\sqrt{3} + 3}\right) = 2$$

$$\sin\left(\theta + \tan^{-1}\frac{2}{2\sqrt{3} + 3}\right) = \frac{2}{\sqrt{25 + 12\sqrt{3}}}$$

$$\alpha = 17.1921...$$

$$17.192....° < \theta + \tan^{-1}\frac{2}{2\sqrt{3} + 3} < 377.192...°$$

$$\theta + \tan^{-1}\frac{2}{2\sqrt{3} + 3} = 180° - \alpha$$

$$\theta = 145.6°$$

Q4

$$\begin{aligned}\sin 2\theta + \cos 2\theta - 1 &= 2\sin\theta\cos\theta + 1 - 2\sin^2\theta - 1\\ &= 2\sin\theta\cos\theta - 2\sin^2\theta\\ &= 2\sin\theta\left(\cos\theta - \sin\theta\right)\\ &= 2\sin\theta\left(\sqrt{2}\cos\left(\theta + \tan^{-1}1\right)\right)\\ &= 2\sin\theta\left(\sqrt{2}\cos\left(\theta + 45°\right)\right)\\ &= 2\sqrt{2}\sin\theta\cos\left(\theta + 45°\right)\end{aligned}$$

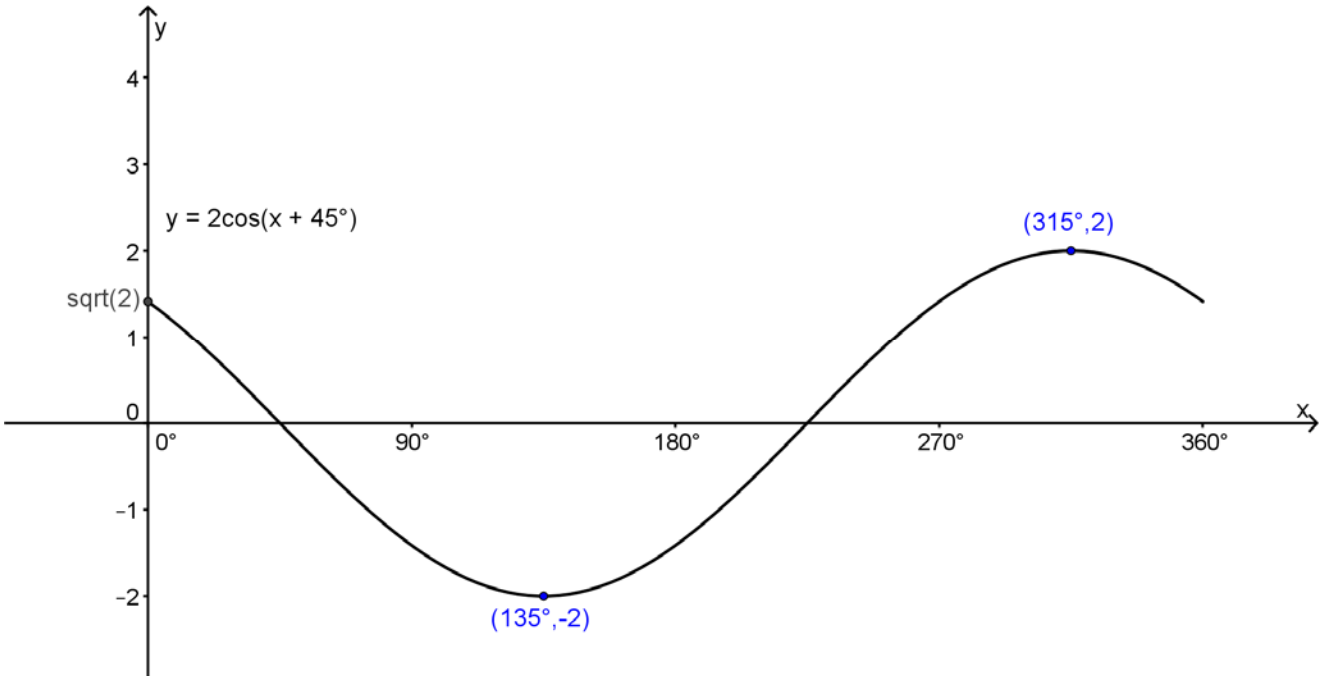
$$2\sqrt{2}\sin\theta\cos\left(\theta + 45°\right) = 0$$

$$\sin\theta = 0 \quad \text{or} \quad \cos\left(\theta + 45°\right) = 0$$

$$\theta = 0°, 180°, 360°, 45°, 225°$$

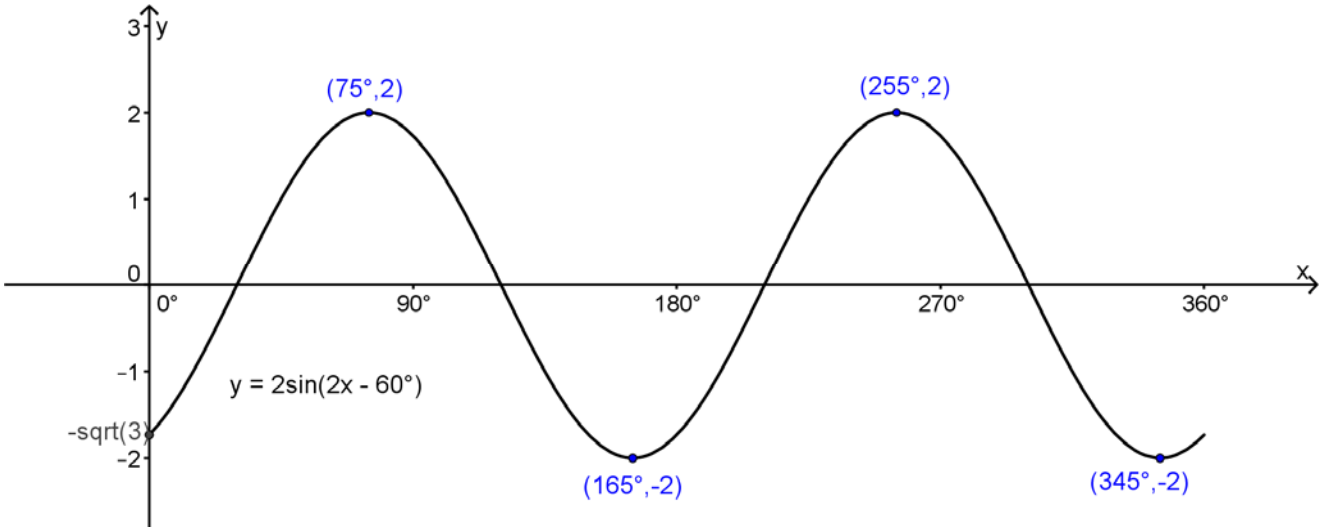
Q5(a)

$$f(x) = \sqrt{2}\cos x - \sqrt{2}\sin x$$
$$= 2\cos(x + 45°)$$



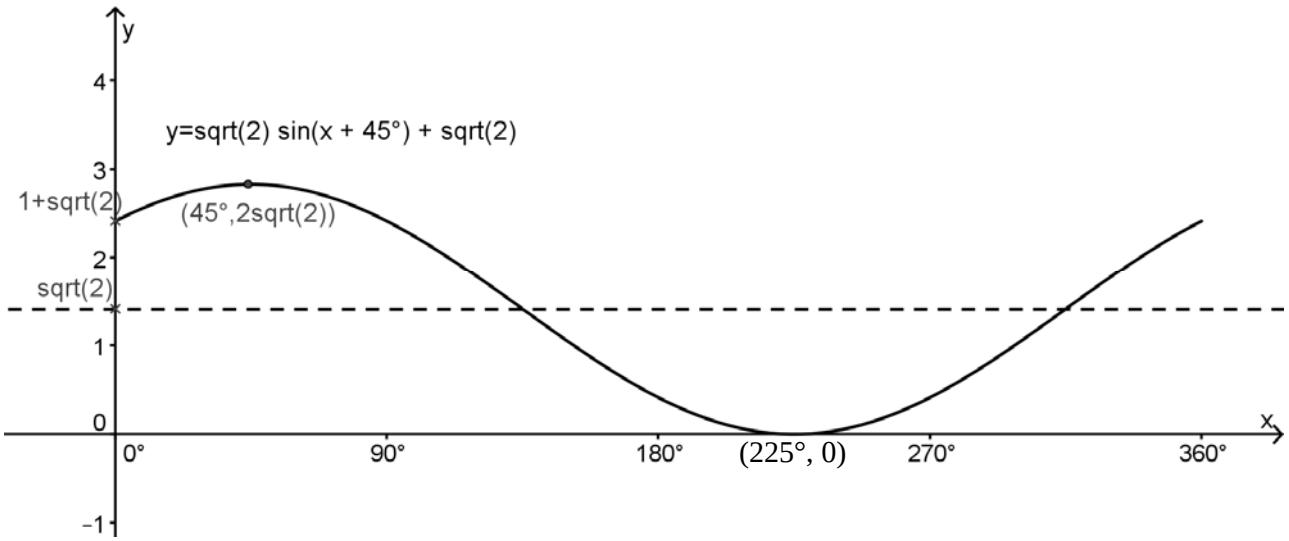
Q5(b)

$$f(x) = \sin 2x - \sqrt{3}\cos 2x$$
$$= 2\sin(2x - 60°)$$



Q5(c)

$$f(x) = \sin x + \cos x + \sqrt{2}$$
$$= \sqrt{2}\sin\left(x + 45^\circ\right) + \sqrt{2}$$



Q6

$$-1 \le \cos x \le 1$$
$$-|a| \le |a|\cos x + b \le |a|$$
$$-|a| + b \le |a|\cos x + b \le |a| + b$$
$$|a| + b = 1 \rightarrow (1)$$
$$-|a| + b = -7 \rightarrow (2)$$
$$\therefore |a| = 4, b = -3$$
$$a = 4 \text{ or } -4, b = -3$$

Similarly,

$$-1 \le \sin x \le 1$$
$$-|ab| \le ab\sin x \le |ab|$$
$$3 - |ab| \le 3 + ab\sin x \le 3 + |ab|$$

Maximum value

$$= 3 + |ab|$$
$$= 3 + |\pm 4 \times -3|$$
$$= 3 + |\pm 12|$$
$$= 15$$