Year 4 Math Assignment 4: Further Trigo

Q1 If A is an obtuse angle and $\sin A = \dfrac{1}{\sqrt{5}}$, find the values of $\sin 2A$, cos2A, sin4A and $\sin \dfrac{A}{2}$.

Q2 Given that $270° \le A \le 360°$, and $\tan \dfrac{A}{2} = -\dfrac{2}{3}$, find $\cos A$, $\tan A$ and $\sin A$.

Q3 Solve the following equations for $0° \le x \le 360°$

(a) $\dfrac{1}{2}\sin x = \sin 2x$

(b) $3\cos 2x + \sin x - 2 = 0$

(c) $\cos 2x + 2\sin^2 \dfrac{x}{2} = 1$

Q4 Prove the following identities

(a) $\cot A - 2\cot 2A = \tan A$

(b) $\cot A + \tan 2A = \cot A \sec 2A$

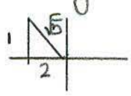
(c) $\dfrac{\cos 2x + \sin 2x - 1}{\cos 2x - \sin 2x + 1} = \tan x$

(d) $\dfrac{1 + \sin 2x}{\cos 2x} = \dfrac{\cos x + \sin x}{\cos x - \sin x}$

Q5 Given that $\dfrac{\sin(A-B)}{\sin(A+B)} = \dfrac{5}{12}$ and $\tan A = \dfrac{3}{4}$. Without using a calculator, find the value of $\tan B$.

Year 4 Assignment 4 Solutions

Q1)



$\sin 2A = 2\left(\frac{1}{\sqrt{5}}\right)\left(-\frac{2}{\sqrt{5}}\right) = -\frac{4}{5}$ #

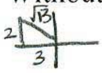
$\cos 2A = 2\left(-\frac{2}{\sqrt{5}}\right)^2 - 1 = \frac{8}{5} - 1 = \frac{3}{5}$ #

$\sin 4A = 2\sin 2A \cos 2A = 2\left(-\frac{4}{5}\right)\left(\frac{3}{5}\right) = -\frac{24}{25}$ #

$\cos A = 1 - 2\sin^2 \frac{A}{2}$

$\sin \frac{A}{2} = \sqrt{\frac{1-\cos A}{2}} = \sqrt{\frac{1-\left(-\frac{2}{\sqrt{5}}\right)}{2}} = \sqrt{\frac{\sqrt{5}+2}{2\sqrt{5}}}$ #

(rej -ve as $\sin \frac{A}{2} > 0$)



$135° \le \frac{A}{2} \le 180°$

Q2a) $\tan \frac{A}{2} = -\frac{2}{3} \Rightarrow \sin \frac{A}{2} = \frac{2}{\sqrt{13}}$

$\cos \frac{A}{2} = -\frac{3}{\sqrt{13}}$

$\therefore \cos A = 2\cos^2 \frac{A}{2} - 1 = 2\left(-\frac{3}{\sqrt{13}}\right)^2 - 1 = \frac{5}{13}$ #

$\tan A = \frac{2\tan \frac{A}{2}}{1 - \tan^2 \frac{A}{2}} = \frac{2\left(-\frac{2}{3}\right)}{1 - \left(-\frac{2}{3}\right)^2} = -\frac{12}{5}$ #

$\sin A = 2\sin \frac{A}{2} \cos \frac{A}{2} = 2\left(\frac{2}{\sqrt{13}}\right)\left(-\frac{3}{\sqrt{13}}\right) = -\frac{12}{13}$ #

Q3a) $\frac{1}{2}\sin x = \sin 2x$

$\frac{1}{2}\sin x = 2\sin x \cos x$

$\sin x \left(2\cos x - \frac{1}{2}\right) = 0$

$\sin x = 0$ or $\cos x = \frac{1}{4}$

B·A $= 0°$ or B·A $= 75.52°$

$x = 0°, 180°, 360°, 75.5°, 284.5°$ #

(b) $3\cos 2x + \sin x - 2 = 0$

$3(1 - 2\sin^2 x) + \sin x - 2 = 0$

$6\sin^2 x - \sin x - 1 = 0$

$(3\sin x + 1)(2\sin x - 1) = 0$

$\sin x = -\frac{1}{3}$ or $\sin x = \frac{1}{2}$

B·A $= 19.47°$ or B·A $= 30°$

$x = 199.5°, 340.5°, 30°, 150°$ #

(c) $\cos 2x + 2\sin^2 \frac{x}{2} = 1$

$2\cos^2 x - 1 = 1 - 2\sin^2 \frac{x}{2}$

$2\cos^2 x - 1 = \cos x$

$2\cos^2 x - \cos x - 1 = 0$

$(2\cos x + 1)(\cos x - 1) = 0$

$\cos x = -\frac{1}{2}$ or $\cos x = 1$

B·A $= 60°$ or B·A $= 0°$

$x = 120°, 240°, 0°, 360°$ #

Q4(a) $\cot A - 2\cot 2A = \tan A$

LHS: $\dfrac{1}{\tan A} - \dfrac{2}{\tan 2A}$

$= \dfrac{1}{\tan A} - \dfrac{2(1-\tan^2 A)}{2\tan A}$

$= \dfrac{1 - 1 + \tan^2 A}{\tan A}$

$= \tan A = RHS$ #

(b) $\cot A + \tan 2A = \cot A \sec 2A$

LHS: $\dfrac{\cos A}{\sin A} + \dfrac{2\sin A\cos A}{\cos^2 A - \sin^2 A}$

$= \dfrac{\cos^3 A - \cos A\sin^2 A + 2\sin^2 A\cos A}{\sin A(\cos^2 A - \sin^2 A)}$

$= \dfrac{\cos^3 A + \sin^2 A\cos A}{\sin A(\cos^2 A - \sin^2 A)}$

$= \dfrac{\cos A(\cos^2 A + \sin^2 A)}{\sin A(\cos 2A)}$

$= \cot A \sec 2A = RHS$ #

(c) $\dfrac{\cos 2x + \sin 2x - 1}{\cos 2x - \sin 2x + 1} = \tan x$

LHS: $\dfrac{1 - 2\sin^2 x + 2\sin x\cos x - 1}{2\cos^2 x - 1 - 2\sin x\cos x + 1}$

$= \dfrac{2\sin x(\cos x - \sin x)}{2\cos x(\cos x - \sin x)}$

$= \tan x = RHS$ #

(d) $\dfrac{1 + \sin 2x}{\cos 2x} = \dfrac{\cos x + \sin x}{\cos x - \sin x}$

LHS: $\dfrac{1 + 2\sin x\cos x}{\cos^2 x - \sin^2 x}$

$= \dfrac{\cos^2 x + \sin^2 x + 2\sin x\cos x}{(\cos x + \sin x)(\cos x - \sin x)}$

$= \dfrac{(\cos x + \sin x)^2}{(\cos x + \sin x)(\cos x - \sin x)}$

$= \dfrac{\cos x + \sin x}{\cos x - \sin x} = RHS$ #

Q5) $\dfrac{\sin(A-B)}{\sin(A+B)} = \dfrac{5}{12}$

$\tan A = \dfrac{3}{4}$

$\dfrac{\sin A\cos B - \cos A\sin B}{\sin A\cos B + \cos A\sin B} = \dfrac{5}{12}$

$12\sin A\cos B - 12\cos A\sin B = 5\sin A\cos B + 5\cos A\sin B$

$7\sin A\cos B = 17\cos A\sin B$

$7\tan A = 17\tan B$

$\Rightarrow \tan B = \dfrac{7\tan A}{17}$

$= \dfrac{21}{68}$ #