Year 4 Math Assignment 4: Further Trigo **Time: 90 mins**

Q1 If A is an obtuse angle and $\sin A = \frac{1}{\sqrt{5}}$, find the values of $\sin 2A$, cos2A, sin4A and $\sin\frac{A}{2}$.

Q2 Given that $270° \le A \le 360°$, and $\tan\frac{A}{2} = -\frac{2}{3}$, find $\cos A$, $\tan A$ and $\sin A$.

Q3 Solve the following equations for $0° \le x \le 360°$

(a) $\frac{1}{2}\sin x = \sin 2x$

(b) $3\cos 2x + \sin x - 2 = 0$

(c) $\cos 2x + 2\sin^2\frac{x}{2} = 1$

Q4 Prove the following identities

(a) $\cot A - 2\cot 2A \equiv \tan A$

(b) $\cot A + \tan 2A \equiv \cot A \sec 2A$

(c) $\frac{\cos 2x + \sin 2x - 1}{\cos 2x - \sin 2x + 1} \equiv \tan x$

(d) $\frac{1 + \sin 2x}{\cos 2x} \equiv \frac{\cos x + \sin x}{\cos x - \sin x}$

(e) $\cot x - \tan y \equiv \frac{\cos(x+y)}{\sin x \cos y}$

(f) $\frac{1 + \tan x}{1 - \tan x} \equiv \tan\left(x + 45°\right)$

Q5 Given that $\frac{\sin(A-B)}{\sin(A+B)} = \frac{5}{12}$ and $\tan A = \frac{3}{4}$. Without using a calculator, find the value of $\tan B$.

Q6 Prove that $\frac{\sin 2A + \cos 2A + 1}{\sin 2A + \cos 2A - 1} \equiv \frac{\tan\left(45° + A\right)}{\tan A}$. Hence show that $\tan 15° = 2 - \sqrt{3}$

Q7 If $220° < x < 360°$, simplify $\sqrt{2 + \sqrt{2 + 2\cos x}}$.

Q8 Let $f(\theta) = 8\cos^4\frac{\theta}{2} + \cos 2\theta - 8\cos\theta$ for $0 \le \theta \le 360°$

(a) Show $f(\theta) = (2\cos\theta - 1)^2$

(b) Find the greatest and least values of $f(\theta)$ and the corresponding values of θ.

(c) Using (b), sketch the graph of $f(\theta)$ for $0 \le \theta \le 360°$.

Q9 Without using a calculator, evaluate $\sin^4 15° + \sin^4 75° + \sin^4 105° + \sin^4 165°$.