Year 4 Math Assignment 3 Solutions

Q1(a) $\cos 75^\circ = \cos(30^\circ + 45^\circ)$

$= \cos 30^\circ \cos 45^\circ - \sin 30^\circ \sin 45^\circ$

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) - \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$

$= \frac{\sqrt{3}-1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6}-\sqrt{2}}{4}$ #

(b) $\sin(-165^\circ) = -\sin 15^\circ$

$= -\sin(45^\circ - 30^\circ)$

$= -\left[\left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)\right]$

$= \frac{1-\sqrt{3}}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}-\sqrt{6}}{4}$ #

(c) $\tan\left(-\frac{\pi}{12}\right) = -\tan\left(\frac{\pi}{12}\right)$

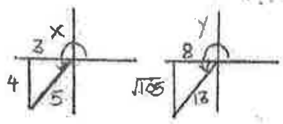
$= -\tan\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$

$= -\left(\frac{\tan\frac{\pi}{4} - \tan\frac{\pi}{6}}{1 + \tan\frac{\pi}{4}\tan\frac{\pi}{6}}\right)$

$= -\left(\frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}}\right)$

$= -\frac{\sqrt{3}-1}{\sqrt{3}+1} = \frac{1-\sqrt{3}}{1+\sqrt{3}} \times \frac{1-\sqrt{3}}{1-\sqrt{3}}$

$= \frac{4-2\sqrt{3}}{-2} = -2+\sqrt{3}$

Q2(a) 

$\cos(x-y) = \left(-\frac{3}{5}\right)\left(-\frac{8}{13}\right) + \left(-\frac{4}{5}\right)\left(-\frac{\sqrt{105}}{13}\right)$

$= \frac{24+4\sqrt{105}}{65}$ #

(b) $\tan(x-y) = \frac{\frac{4}{3} - \frac{\sqrt{105}}{8}}{1 + \left(\frac{4}{3}\right)\left(\frac{\sqrt{105}}{8}\right)} = \frac{32-3\sqrt{105}}{24+4\sqrt{105}} \times \frac{24-4\sqrt{105}}{24-4\sqrt{105}}$

$= \frac{2028-200\sqrt{105}}{-1104}$

$= \frac{507-50\sqrt{105}}{-276}$ #

(c) $\sin(x+y) = \left(-\frac{4}{5}\right)\left(-\frac{8}{13}\right) + \left(-\frac{3}{5}\right)\left(-\frac{\sqrt{105}}{13}\right)$

$= \frac{32+3\sqrt{105}}{65}$ #

Q3(a) $\sin x = \cos(x+30^\circ)$

$\sin x = \cos x\left(\frac{\sqrt{3}}{2}\right) - \sin x\left(\frac{1}{2}\right)$

$\frac{3}{2}\sin x = \frac{\sqrt{3}}{2}\cos x$

$\tan x = \frac{\sqrt{3}}{3}$

B.A $= 30^\circ$

$\therefore x = 30^\circ, 210^\circ$ #

3(b) $\sin(x+30^\circ) - \cos x = 0$

$\sin x\left(\frac{\sqrt{3}}{2}\right) + \cos x\left(\frac{1}{2}\right) - \cos x = 0$

$\frac{\sqrt{3}}{2}\sin x = \frac{1}{2}\cos x$

$\tan x = \frac{1}{\sqrt{3}}$

B.A $= 30^\circ$

$\therefore x = 30^\circ, 210^\circ$ #

(c) $7\tan x = 4\tan(45^\circ - x)$

$7\tan x = 4\left(\frac{\tan 45^\circ - \tan x}{1 + \tan x}\right)$

$7\tan x + 7\tan^2 x = 4 - 4\tan x$

$7\tan^2 x + 11\tan x - 4 = 0$

$\tan x = 0.30459\ldots$ or $\tan x = -1.876\ldots$

B.A $= 16.94^\circ$ or B.A $= 61.94^\circ$

$x = 16.9^\circ, 196.9^\circ, 118.1^\circ, 298.1^\circ$ #

Q4) $\frac{\tan A + \tan B}{1 - \tan A \tan B} = \sqrt{3}$

$\tan(A+B) = \sqrt{3}$

B.A $= 60^\circ$

$A+B = 60^\circ$ # (rej 240°)

Q5(a) $\frac{\cos 3A - \cos 2A}{\sin 3A + \sin 2A} \equiv -\tan\frac{1}{2}A$

LHS: $\frac{\cos(2\frac{1}{2}A + \frac{1}{2}A) - \cos(2\frac{1}{2}A - \frac{1}{2}A)}{\sin(2\frac{1}{2}A + \frac{1}{2}A) + \sin(2\frac{1}{2}A - \frac{1}{2}A)}$

$= \frac{(\cos 2\frac{1}{2}A \cos\frac{1}{2}A - \sin 2\frac{1}{2}A \sin\frac{1}{2}A) - (\cos 2\frac{1}{2}A \cos\frac{1}{2}A - \sin 2\frac{1}{2}A \sin\frac{1}{2}A)}{(\sin 2\frac{1}{2}A \cos\frac{1}{2}A + \cos 2\frac{1}{2}A \sin\frac{1}{2}A) + (\sin 2\frac{1}{2}A \cos\frac{1}{2}A - \cos 2\frac{1}{2}A \sin\frac{1}{2}A)}$

$= \frac{-2\sin 2\frac{1}{2}A \sin\frac{1}{2}A}{2\sin 2\frac{1}{2}A \cos\frac{1}{2}A}$

$= -\tan\frac{1}{2}A =$ RHS #

(b) $\frac{\sin(A+B)}{\sin(A-B)} \equiv \frac{\tan A + \tan B}{\tan A - \tan B}$

LHS: $\frac{\sin A \cos B + \cos A \sin B}{\sin A \cos B - \cos A \sin B}$ (divide each term by $\cos A \cos B$)

$= \frac{\tan A + \tan B}{\tan A - \tan B}$

$=$ RHS #

(c) $\cot(A+B) \equiv \frac{\cot A \cot B - 1}{\cot A + \cot B}$

LHS: $\frac{1}{\tan(A+B)} = \frac{1 - \tan A \tan B}{\tan A + \tan B}$

$= \frac{\cot A \cot B - 1}{\cot B + \cot A}$ (divide each term by $\tan A \tan B$)

$=$ RHS #

Q6) Let the squares be of length 1 unit.

$\tan\gamma = 1$, $\tan\beta = \frac{1}{2}$, $\tan\alpha = \frac{1}{3}$

$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$= \frac{\frac{1}{3} + \frac{1}{2}}{1 - (\frac{1}{3})(\frac{1}{2})} = 1$

$\tan(\alpha+\beta) = \tan\gamma \Rightarrow \alpha + \beta = \gamma$ #