Year 4 Math Assignment 25: Vectors

Q1 Given that $\underset{\sim}{a} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\underset{\sim}{b} = \begin{pmatrix} \lambda \\ 3 \end{pmatrix}$, determine the value(s) of λ such that

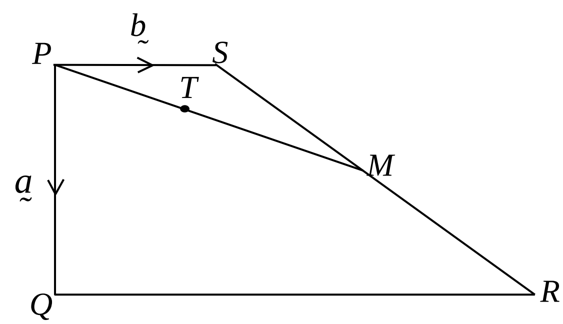
(a) $\underset{\sim}{a}$ is parallel to $\underset{\sim}{b}$,

(b) the magnitude of $\underset{\sim}{a} + \underset{\sim}{b}$ is 5 units.

Q2 The points *A*, *B* and *C* have position vectors $\underset{\sim}{p} + 2\underset{\sim}{q}$, $3\underset{\sim}{p} - \underset{\sim}{q}$ and $5\underset{\sim}{p} + \lambda\underset{\sim}{q}$ respectively to the origin.

Find the value of λ for which *A*, *B* and *C* are collinear.

Q3 In the diagram, *PQRS* is a quadrilateral with $\overrightarrow{PQ} = \underset{\sim}{a}$, $\overrightarrow{PS} = \underset{\sim}{b}$, $\overrightarrow{QR} = 3\overrightarrow{PS}$ and *M* is the midpoint of *SR*. The ratio of *PT* : *TM* is 2 : 3.



(a) Find the following vectors in terms of $\underset{\sim}{a}$ and/or $\underset{\sim}{b}$

(i) $\overrightarrow{SR}$ (ii) $\overrightarrow{SM}$ (iii) $\overrightarrow{PM}$ (iv) $\overrightarrow{PT}$

(b) Show that $\overrightarrow{QT} = \mu\overrightarrow{TS}$ where μ is a constant to be determined.

(c) Find the ratio of the area of ΔSMT : area of ΔPQT.

Q4 Three points *A*, *B* and *C* have position vectors $\begin{pmatrix} 6 \\ 7 \end{pmatrix}$, $\begin{pmatrix} 0 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$ respectively. *P* is the point on the line *BC* such that $\overrightarrow{BP} = \frac{3}{2}\overrightarrow{PC}$, and *Q* is the point on *BC* produced such that $\overrightarrow{BQ} = \frac{3}{2}\overrightarrow{CQ}$. Find the position vector of *P* and of *Q*.

Q5 The position vectors of the points *A* and *B*, with respect to an origin *O*, are $\underset{\sim}{a}$ and $\underset{\sim}{b}$ respectively. Given that $\underset{\sim}{a} = 7\underset{\sim}{i} + 2\underset{\sim}{j}$ and $\underset{\sim}{b} = \underset{\sim}{i} + 8\underset{\sim}{j}$, find the unit vector which is perpendicular to $\underset{\sim}{a} - 3\underset{\sim}{b}$.

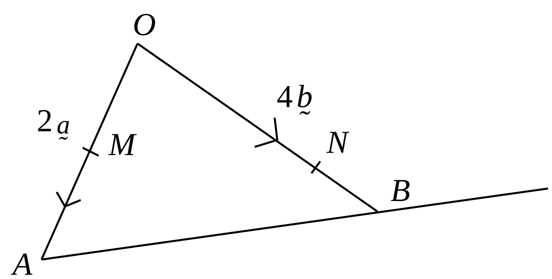
Q6 Points A, B and C have position vectors $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$ respectively, relative to an origin O. The point P lies on BC such that $BP : PC = 1 : 2$. The point Q lies on AP produced such that $AP : PQ = 1 : 2$. Find, in terms of $\underset{\sim}{a}$, $\underset{\sim}{b}$, and/or $\underset{\sim}{c}$.

(i) $\overrightarrow{OP}$

(ii) $\overrightarrow{OQ}$

Hence, show that $\overrightarrow{CQ}$ is parallel to $\overrightarrow{AB}$.

Q7



In the diagram, OAB is a triangle with $\overrightarrow{OA} = 2\underset{\sim}{a}$ and $\overrightarrow{OB} = 4\underset{\sim}{b}$. The line AB is produced to the point C such that $AB = 2BC$.

(i) Find $\overrightarrow{AB}$, $\overrightarrow{BC}$ and $\overrightarrow{OC}$ in terms of $\underset{\sim}{a}$ and/or $\underset{\sim}{b}$.

(ii) Given that M is the mid-point of OA and N is a point on OB such that $ON : NB = 3 : 1$, show that the points M, N and C are collinear.

(iii) If K is a point on AB such that MK is parallel to OB, find the numerical value of $\dfrac{AC}{AK}$.

(iv) The area of ΔNBC is 48cm^2. Calculate the area of
(i) ΔMKC,
(ii) ΔMAK.

Q8 With respect to the origin O, the points P and Q have variable position vectors $\underset{\sim}{p}$ and $\underset{\sim}{q}$ respectively, given by $\underset{\sim}{p} = (\cos t)\underset{\sim}{i} + (\sin t)\underset{\sim}{j}$ and $\underset{\sim}{q} = (\cos 2t)\underset{\sim}{i} - (\sin 2t)\underset{\sim}{j}$, where $0 \le t \le 2\pi$. Show that $\underset{\sim}{p} \cdot \underset{\sim}{q} = \cos 3t$ and hence or otherwise, find the greatest value of the angle POQ.