**Year 4 Math Assignment 25 solutions**

**Q1(i)**

$$\underset{\sim}{a} = k\underset{\sim}{b}$$

$$\begin{pmatrix}2\\1\end{pmatrix} = k\begin{pmatrix}\lambda\\3\end{pmatrix}$$

$$k = \frac{1}{3}$$

$$\frac{1}{3}\lambda = 2$$

$$\lambda = 6$$

(ii)

$$\begin{pmatrix}2\\1\end{pmatrix} + \begin{pmatrix}\lambda\\3\end{pmatrix} = \begin{pmatrix}2+\lambda\\4\end{pmatrix}$$

$$\sqrt{(2+\lambda)^2 + 16} = 5$$

$$4 + 4\lambda + \lambda^2 = 25 - 16$$

$$\lambda^2 + 4\lambda^2 - 5 = 0$$

$$(\lambda+5)(\lambda-1) = 0$$

$$\lambda = 1 \text{ or } -5$$

**Q2**

$$\overrightarrow{OA} = \underset{\sim}{p} + 2\underset{\sim}{q} = \begin{pmatrix}1\\2\end{pmatrix}$$

$$\overrightarrow{OB} = 3\underset{\sim}{p} - \underset{\sim}{q} = \begin{pmatrix}3\\-1\end{pmatrix}$$

$$\overrightarrow{OC} = 5\underset{\sim}{p} + \lambda\underset{\sim}{q} = \begin{pmatrix}5\\\lambda\end{pmatrix}$$

$$\overrightarrow{AB} = \begin{pmatrix}3\\-1\end{pmatrix} - \begin{pmatrix}1\\2\end{pmatrix} = \begin{pmatrix}2\\-3\end{pmatrix}$$

$$\overrightarrow{AC} = \begin{pmatrix}5\\\lambda\end{pmatrix} - \begin{pmatrix}1\\2\end{pmatrix} = \begin{pmatrix}4\\\lambda-2\end{pmatrix}$$

$$\overrightarrow{AB} = k\,\overrightarrow{AC} \quad [A, B, C \text{ are collinear}]$$

$$\begin{pmatrix}2\\-3\end{pmatrix} = k\begin{pmatrix}4\\\lambda-2\end{pmatrix}$$

$$\therefore k = \frac{1}{2}, \lambda = -4$$

**Q3**

(a)(i)

$$\overrightarrow{SR} = \underset{\sim}{a} + 2\underset{\sim}{b}$$

(a)(ii)

$$\overrightarrow{SM} = 0.5\overrightarrow{SR}$$

$$\overrightarrow{SM} = 0.5\underset{\sim}{a} + \underset{\sim}{b}$$

(a)(iii)

$$\overrightarrow{PM} = \overrightarrow{PS} + \overrightarrow{SM}$$

$$\overrightarrow{PM} = 0.5\underset{\sim}{a} + 2\underset{\sim}{b}$$

(a) (iv)

$$\overrightarrow{PM} = 0.5\underset{\sim}{a} + 2\underset{\sim}{b}$$

**Q3**

(b)

$$\overrightarrow{QS} = \overrightarrow{PS} - \overrightarrow{PQ}$$

$$\overrightarrow{QS} = \underset{\sim}{b} - \underset{\sim}{a}$$

$$\overrightarrow{PT} + \overrightarrow{TS} = \overrightarrow{PS}$$

$$\overrightarrow{TS} = \frac{1}{5}\underset{\sim}{b} - \frac{1}{5}\underset{\sim}{a}$$

$$\overrightarrow{QS} = 5\overrightarrow{TS}$$

(c) Area of $\Delta SMT$ : Area of $\Delta PQT$

1 : 4

| Q4 | Q5 |
|---|---|
| $\overrightarrow{OA}=\begin{pmatrix}6\\7\end{pmatrix}$ <br> $\overrightarrow{OB}=\begin{pmatrix}0\\4\end{pmatrix}$ <br> $\overrightarrow{OA}=\begin{pmatrix}4\\3\end{pmatrix}$ <br> $\overrightarrow{BP}=\frac{3}{2}\overrightarrow{PC}$ <br> $\overrightarrow{OP}-\overrightarrow{OB}=\frac{3}{2}\left(\overrightarrow{OC}-\overrightarrow{OP}\right)$ <br> $\frac{5}{2}\overrightarrow{OP}=\frac{3}{2}\overrightarrow{OC}+\overrightarrow{OB}$ <br> $\overrightarrow{OP}=\frac{3}{5}\overrightarrow{OC}+\frac{2}{5}\overrightarrow{OB}$ <br> $=\frac{3}{5}\begin{pmatrix}4\\3\end{pmatrix}+\frac{2}{5}\begin{pmatrix}0\\4\end{pmatrix}$ <br> $=\begin{pmatrix}\frac{14}{5}\\\frac{17}{5}\end{pmatrix}$ <br> $\overrightarrow{BQ}=\frac{3}{2}\overrightarrow{CQ}$ <br> $\overrightarrow{OQ}-\overrightarrow{OB}=\frac{3}{2}\left(\overrightarrow{OQ}-\overrightarrow{OC}\right)$ <br> $\frac{1}{2}\overrightarrow{OQ}=\frac{3}{2}\overrightarrow{OC}+\overrightarrow{OB}$ <br> $\overrightarrow{OQ}=3\overrightarrow{OC}+2\overrightarrow{OB}$ <br> $=3\begin{pmatrix}4\\3\end{pmatrix}+2\begin{pmatrix}0\\4\end{pmatrix}$ <br> $=\begin{pmatrix}12\\15\end{pmatrix}$ | $\underset{\sim}{a}-3\underset{\sim}{b}=7\underset{\sim}{i}+2\underset{\sim}{j}-3\left(\underset{\sim}{i}+8\underset{\sim}{j}\right)$ <br> $=4\underset{\sim}{i}-22\underset{\sim}{j}$ <br> $=\begin{pmatrix}4\\-22\end{pmatrix}$ <br> Let vector perpendicular to $\underset{\sim}{a}-3\underset{\sim}{b}$ be $x\underset{\sim}{i}+y\underset{\sim}{j}$ <br> $x\underset{\sim}{i}+y\underset{\sim}{j}=\begin{pmatrix}-22\\-4\end{pmatrix}$ or $\begin{pmatrix}22\\4\end{pmatrix}$ <br> Check using dot product: <br> $\begin{pmatrix}4\\-22\end{pmatrix}\cdot\begin{pmatrix}-22\\-4\end{pmatrix}=0,\quad \begin{pmatrix}4\\-22\end{pmatrix}\cdot\begin{pmatrix}22\\4\end{pmatrix}=0$ <br> Unit Vector = $\frac{1}{\sqrt{4^2+22^2}}\begin{pmatrix}-22\\-4\end{pmatrix}$ or $\frac{1}{\sqrt{4^2+22^2}}\begin{pmatrix}22\\4\end{pmatrix}$ <br> $\frac{1}{5\sqrt{5}}\begin{pmatrix}-11\\-2\end{pmatrix}$ or $\frac{1}{5\sqrt{5}}\begin{pmatrix}11\\2\end{pmatrix}$ |

| Q6 | Q7 |
|---|---|
| $\dfrac{BP}{PC}=\dfrac{1}{2}\to PC=2BP$<br>$\dfrac{AP}{PQ}=\dfrac{1}{2}\to PQ=2AP$<br><br>(i)<br>$\vec{OC}-\vec{OP}=2\left(\vec{OP}-\vec{OB}\right)$<br>$3\vec{OP}=\vec{OC}+2\vec{OB}$<br>$\vec{OP}=\dfrac{1}{3}\underset{\sim}{c}+\dfrac{2}{3}\underset{\sim}{b}$<br>(ii)<br>$\vec{OQ}-\vec{OP}=2\left(\vec{OP}-\vec{OA}\right)$<br>$3\vec{OP}=\vec{OQ}+2\vec{OA}$<br>$\vec{OQ}=3\left(\dfrac{1}{3}\underset{\sim}{c}+\dfrac{2}{3}\underset{\sim}{b}\right)-2\underset{\sim}{a}$<br>$=\underset{\sim}{c}+2\underset{\sim}{b}-2\underset{\sim}{a}$<br>Hence:<br>$\vec{CQ}=2\underset{\sim}{b}-2\underset{\sim}{a}$<br>$=2\left(\underset{\sim}{b}-\underset{\sim}{a}\right)$<br>$=2\vec{AB}\to CQ\,/\!/\,AB$ | Q7<br>(i)<br>$\vec{AB}=\vec{OB}-\vec{OA}$<br>$=4\underset{\sim}{b}-2\underset{\sim}{a}$<br><br>$\vec{BC}=\dfrac{1}{2}\vec{AB}$<br>$=2\underset{\sim}{b}-\underset{\sim}{a}$<br><br>$\vec{OC}=\vec{OB}+\vec{BC}$<br>$=4\underset{\sim}{b}+2\underset{\sim}{b}-\underset{\sim}{a}$<br>$=6\underset{\sim}{b}-\underset{\sim}{a}$<br><br>(ii)<br>$\vec{OM}=\underset{\sim}{a}$<br>$\vec{ON}=\dfrac{3}{4}\vec{OB}=3\underset{\sim}{b}$<br>$\vec{MN}=3\underset{\sim}{b}-\underset{\sim}{a}$<br>$\vec{NC}=6\underset{\sim}{b}-\underset{\sim}{a}-3\underset{\sim}{b}$<br>$=3\underset{\sim}{b}-\underset{\sim}{a}$<br>$\vec{MN}=\vec{NC}\to M,N,C$ are collinear<br><br>(iii)<br>$\vec{MK}=2\underset{\sim}{b}$<br>$\dfrac{AC}{AK}=3$<br><br>(iv)<br>$\triangle MKC\sim\triangle NBC\to\dfrac{\text{Area }\triangle NBC}{\text{Area }\triangle MKC}=\left(\dfrac{1}{2}\right)^2$<br>Area $\triangle MKC=192\text{ cm}^2$<br><br>$\triangle MAK$ and $\triangle NBC$ have common base<br>$\dfrac{\text{Area }\triangle NBC}{\text{Area }\triangle MKC}=\dfrac{NB}{MK}=\dfrac{1}{2}$<br>Area $\triangle MKC=96\text{ cm}^2$ |

Q8

$$\underset{\sim}{p} = (\cos t)\underset{\sim}{i} + (\sin t)\underset{\sim}{j}$$

$$= \begin{pmatrix} \cos t \\ \sin t \end{pmatrix}$$

$$\underset{\sim}{q} = (\cos 2t)\underset{\sim}{i} - (\sin 2t)\underset{\sim}{j}$$

$$= \begin{pmatrix} \cos 2t \\ -\sin 2t \end{pmatrix}$$

$$\underset{\sim}{p} \cdot \underset{\sim}{q} = \begin{pmatrix} \cos t \\ \sin t \end{pmatrix} \cdot \begin{pmatrix} \cos 2t \\ -\sin 2t \end{pmatrix}$$

$$= \cos t \cos 2t - \sin t \sin 2t$$

$$= \cos 3t$$

$$\underset{\sim}{p} \cdot \underset{\sim}{q} = \left|\underset{\sim}{p}\right|\left|\underset{\sim}{q}\right| \cos \angle POQ$$

$$\cos \angle POQ = \frac{\underset{\sim}{p} \cdot \underset{\sim}{q}}{\left|\underset{\sim}{p}\right|\left|\underset{\sim}{q}\right|}$$

$$= \frac{\cos 3t}{\sqrt{\cos^2 t + \sin^2 t}\sqrt{\cos^2 2t + \sin^2 2t}}$$

$$= \cos 3t$$

For greatest $\angle POQ$ ,

$$\cos 3t = -1$$

$$t = \pi$$