Year 4 Math Assignment 24 solutions

Q1

$$s = 2t + \frac{18}{t+1}$$

$$v = 2 - \frac{18}{(t+1)^2}$$

$$a = \frac{36}{(t+1)^3}$$

(i) When $t = 0$, $a = 36\ ms^{-2}$

(ii) When $t = 0$, $s = 18$ m

$$2t + \frac{18}{t+1} = 18$$

$$2t^2 + 2t + 18 = 18t + 18$$

$$2t^2 - 16t = 0$$

$$t(t-8) = 0$$

$$t = 0 \text{ or } t = 8$$

When $t = 8$,

$$v = 2 - \frac{18}{(8+1)^2}$$

$$= \frac{16}{9}$$

$$\approx 1.78 ms^{-1}$$

(iii) $v = 2 - \dfrac{18}{(t+1)^2} = 0$

$$2 = \frac{18}{(t+1)^2}$$

$$(t+1)^2 = 9$$

$$t = 2s$$

(iv) When $t = 0$, $s = 18$ m
When $t = 2$, $s = 10$ m
When $t = 5$, $s = 13$ m

Total distance = 8 + 3 = 11 m

Q2

(i) Particle is at instantaneous rest $\Rightarrow$ $v = 0$

$6 \cos 2t = 0$

$t = \frac{\pi}{4}$ or $t = \frac{3\pi}{4}$

(ii)

$$s = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} 6\cos 2t \ \mathrm{d}t$$

$$= \left| \left[\frac{6\sin 2t}{2} \right]_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \right|$$

$$= 6 \text{ m}$$

Q3

(i)

$$v = t^2 - 14t + 48$$

$$\frac{\mathrm{d}v}{\mathrm{d}t} = 2t - 14 = 0 \rightarrow t = 7$$

$$v = -1 \text{ m/s} \quad \left[\frac{\mathrm{d}^2 v}{\mathrm{d}t^2} = 2 > 0 \rightarrow v \text{ is min} \right]$$

(ii)

$$v = t^2 - 14t + 48 = 0$$

$(t-6)(t-8) = 0 \rightarrow t = 6s$;

(iii)

$$s = \frac{1}{3}t^3 - 7t^2 + 48t + c$$

$$t = 0, s = 0 \rightarrow c = 0$$

$$s = \frac{1}{3}t^3 - 7t^2 + 48t$$

Distance =

$$\left| \left[\frac{1}{3}t^3 - 7t^2 + 48t \right]_0^6 \right| + \left| \left[\frac{1}{3}t^3 - 7t^2 + 48t \right]_6^7 \right| = 108 + \frac{2}{3}$$

$$= 108\frac{2}{3} m$$

(iv)

$$s = 0 \rightarrow t\left(\frac{1}{3}t^2 - 7t + 48\right) = 0$$

$$t^2 - 21t + 144 = 0$$

Since $(-21)^2 - 4(1)(144) < 0$,

$\therefore$ there are no other real values of t

Hence, particle does not return to O.

Q4	Q5
$v = mt + nt^3$ $a = m + 3nt^2$ $a = 0,\ t = 2:$ $m + 12n = 0 \to (1)$ $s = \frac{mt^2}{2} + \frac{nt^4}{4} \quad [t = 0,\ s = 0,\ c = 0]$ $\left[\frac{mt^2}{2} + \frac{nt^4}{4}\right]_2^3 = \frac{11}{4}$ $\frac{9m}{2} + \frac{81n}{4} - 2m - 4n = \frac{11}{4}$ $18m + 81n - 8m - 16n = 11$ $10m + 65n = 11 \to (2)$ Solving (1) and (2): $m = \frac{12}{5},\ n = -\frac{1}{5}$	(i) $v = t^2 - 8t + 7$ $x = \frac{t^3}{3} - 4t^2 + 7t + c$ $t = 0,\ x = 0 \to c = 0$ $x = \frac{t^3}{3} - 4t^2 + 7t$ (ii) $v = t^2 - 8t + 7 = 0$ $t = 1$ or 7 $t = 1: x = \frac{10}{3}$ m $t = 7: x = -\frac{98}{3}$ m $\therefore$ Distance $AB = \frac{10}{3} + \frac{98}{3} = 36$ m (iii) $\left[\frac{t^3}{3} - 4t^2 + 7t\right]_0^1 + \left\|\left[\frac{t^3}{3} - 4t^2 + 7t\right]_1^7\right\| + \left[\frac{t^3}{3} - 4t^2 + 7t\right]_7^9$ $= \frac{10}{3} + 36 + \left[-18 + \frac{98}{3}\right]$ $= 54$ m (iv) $v = t^2 - 8t + 7 = 0$ $a = 2t - 8 \to t = 4\text{s}$ $OC = \left\|\left[\frac{t^3}{3} - 4t^2 + 7t\right]_0^4\right\|$ $= \frac{44}{3}$ m $BC = \left\|\left[\frac{t^3}{3} - 4t^2 + 7t\right]_4^7\right\|$ $= \left\|-\frac{98}{3} - \left(-\frac{44}{3}\right)\right\|$ $= 18$ m Particle is nearer to O.