

Year 4 Math Assignment 21: Integration

1. Integrate

(a) $\int e^{-3x+1} dx$

(b) $\int (e^{2x} + e^{-x})^2 dx$

(c) $\int e^{x+1} - e^{1-x} dx$

(d) $\int (e^{-2x} + 1)^3 dx$

(e) $\int 2e^{\frac{x}{2}+2} dx$

(f) $\int \frac{2}{e^{4x+1}} dx$

2. Integrate

(a) $\int \frac{3}{3x+1} dx$

(b) $\int \frac{x^2 + 2x}{x^3 + 3x^2 - 2} dx$

(c) $\int \frac{x^4 + 3x^2}{x^3} dx$

(d) $\int \tan 2x dx$

(e) $\int \frac{\sec^2 x}{5 + \tan x} dx$

(f) $\int \frac{1 - \sin 2x}{x - \sin^2 x} dx$

3. Find $\frac{d}{dx} \ln(x^2 + 1)$ and hence find an expression for $\int \frac{3x}{x^2 + 1} dx$.

4. Find $\frac{d}{dx} e^{x^2}$ and hence find an expression for $\int x e^{x^2} dx$.

5. A curve has a gradient of 4 at the point (0, 1) and $\frac{d^2y}{dx^2} = 3e^{3x} - e^{-x}$. Find the equation of the curve.

6. Show that $\frac{2}{x+1} + \frac{5}{x+3} = \frac{7x+11}{x^2+4x+3}$. Hence, find $\int \frac{28x+44}{x^2+4x+3} dx$.

7. Use long division to show $\frac{x^2+2x+1}{x+5} = x-3 + \frac{16}{x+5}$. Hence find an expression for $\int \frac{x^2+2x+1}{x+5} dx$

8. (a) Find an expression for $f(x)$ such that $\frac{d}{dx}[f(x)] = \frac{1}{x \ln x}$ and hence evaluate $\int_2^4 \frac{1}{x \ln x} dx$.

(b) Find $\frac{d}{dx} x^n \ln x$, where n is a positive integer, and hence find the integrals $\int \ln x dx$, $\int x \ln x dx$, $\int x^2 \ln x dx$ and $\int x^m \ln x dx$, where m is a positive integer.