Assignment 21 solutions

1(a) $\int e^{-3x+1}\ dx = -\frac{1}{3}e^{-3x+1} + c$

1(b)

$$\int\left(e^{2x}+e^{-x}\right)^2\ dx = \int e^{4x}+2e^{x}+e^{-2x}\ dx$$
$$= \frac{1}{4}e^{4x} + 2e^{x} - \frac{1}{2}e^{-2x} + c$$

1(c) $\int e^{x+1} - e^{1-x}\ dx = e^{x+1} + e^{1-x} + c$

1(d)

$$\int\left(e^{-2x}+1\right)^3\ dx = \int e^{-6x}+3e^{-4x}+3e^{-2x}+1\ dx$$
$$= -\frac{1}{6}e^{-6x} - \frac{3}{4}e^{-4x} - \frac{3}{2}e^{-2x} + x + c$$

1(e) $\int 2e^{\frac{x}{2}+2}\ dx = 4e^{\frac{x}{2}+2} + c$

1(f)

$$\int \frac{2}{e^{4x+1}}\ dx = \int 2e^{-4x-1}\ dx$$
$$= -\frac{1}{2}e^{-4x-1} + c$$

2(a) $\int \frac{3}{3x+1}\ dx = \ln\left|3x+1\right| + c$

2(b)

$$\int \frac{x^2+2x}{x^3+3x^2-2}\ dx = \frac{1}{3}\int \frac{3\left(x^2+2x\right)}{x^3+3x^2-2}\ dx$$
$$= \frac{1}{3}\ln\left|x^3+3x^2-2\right|$$

2(c)

$$\int \frac{x^4+3x^2}{x^3}\ dx = \int x + \frac{3}{x}\ dx$$
$$= \frac{x^2}{2} + 3\ln\left|x\right| + c$$

2(d)

$$\int \tan 2x\ dx = -\frac{1}{2}\int \frac{-2\sin 2x}{\cos 2x}\ dx$$
$$= -\frac{1}{2}\ln\left|\cos 2x\right| + c$$

2(e) $\int \frac{\sec^2 x}{5+\tan x}\ dx = \ln\left|5+\tan x\right| + c$

2(f)

$$\int \frac{1-\sin 2x}{x-\sin^2 x}\ dx = \int \frac{1-2\sin x\cos x}{x-\sin^2 x}\ dx$$
$$= \ln\left|x-\sin^2 x\right| + c$$

3

$$\frac{d}{dx}\ln(x^2+1) = \frac{2x}{x^2+1}$$

$$\int \frac{3x}{x^2+1}dx = \frac{3}{2}\int \frac{2x}{x^2+1}dx$$
$$= \frac{3}{2}\ln\left|x^2+1\right| + c$$

4

$$\frac{d}{dx}e^{x^2} = 2xe^{x^2}$$

$$\int xe^{x^2}\ dx = \frac{1}{2}\int 2xe^{x^2}\ dx$$
$$= \frac{1}{2}e^{x^2} + c$$

5

$$\frac{d^2y}{dx^2} = 3e^{3x} - e^{-x}$$
$$\frac{dy}{dx} = \int 3e^{3x} - e^{-x}\ dx = e^{3x} + e^{-x} + c$$
$$4 = 1 + 1 + c \rightarrow c = 2$$
$$\frac{dy}{dx} = e^{3x} + e^{-x} + 2$$
$$y = \int e^{3x} + e^{-x} + 2\ dx = \frac{1}{3}e^{3x} - e^{-x} + 2x + d$$
$$1 = \frac{1}{3} - 1 + 0 + d \rightarrow d = \frac{5}{3}$$
$$y = \frac{1}{3}e^{3x} - e^{-x} + 2x + \frac{5}{3}$$

6

$$\frac{2}{x+1}+\frac{5}{x+3}=\frac{2(x+3)+5(x+1)}{(x+1)(x+3)}$$
$$=\frac{7x+11}{x^2+4x+3}$$

$$\int\frac{28x+44}{x^2+4x+3}\,dx=\int\frac{4(7x+11)}{x^2+4x+3}\,dx$$
$$=4\int\frac{2}{x+1}+\frac{5}{x+3}\,dx$$
$$=8\ln|x+1|+20\ln|x+3|+c$$

7.

$$\frac{x^2+2x+1}{x+5}=x-3+\frac{16}{x+5}$$

$$\begin{array}{r|l} & x-3 \\ x+5 & x^2+2x+1 \\ & x^2+5x \\ & -3x+1 \\ & -3x-15 \\ & 16 \end{array}$$

$$\int\frac{x^2+2x+1}{x+5}\,dx=\int x-3+\frac{16}{x+5}\,dx$$
$$=\frac{x^2}{2}-3x+16\ln|x+5|+c$$

8(a).

$$\frac{d}{dx}[f(x)]=\frac{1}{x\ln x}\rightarrow f(x)=\ln(\ln x)$$
$$\int_2^4\frac{1}{x\ln x}\,dx=\left[\ln(\ln x)\right]_2^4$$
$$=\ln(\ln 4)-\ln(\ln 2)$$
$$=\ln\frac{\ln 2^2}{\ln 2}$$
$$=\ln 2$$

8(b).

$$\frac{d}{dx}x^n\ln x=\frac{x^n}{x}+nx^{n-1}\ln x$$

$$x^n\ln x=\int x^{n-1}+nx^{n-1}\ln x\,dx$$

$$\int\ln x\,dx=x\ln x-x+c\quad\text{where } n=1$$

$$n=2\rightarrow\int x\ln x\,dx=\frac{1}{2}\left(x^2\ln x-\frac{x^2}{2}\right)+c$$

$$n=3\rightarrow\int x^2\ln x\,dx=\frac{1}{3}\left(x^3\ln x-\frac{x^3}{3}\right)+c$$

$$n=m+1\rightarrow\int x^m\ln x\,dx=\frac{1}{m+1}\left(x^{m+1}\ln x-\frac{x^{m+1}}{m+1}\right)+c$$