Assignment 20 solutions

$$\cos 2x = 1 - 2\sin^2 x$$
$$= 2\cos^2 x - 1$$

where c is a const.

1a) $\int 8\sec^2 5x \, dx = \dfrac{8\tan 5x}{5} + c \; \#$

b) $\int \cos\frac{x}{3} \, dx = 3\sin\frac{x}{3} + c \; \#$

c) $\int x^2 + \sec^2\frac{\pi}{2}x \, dx = \dfrac{x^3}{3} + \frac{2}{\pi}\tan\frac{\pi}{2}x + c \; \#$

d) $\int 2\sin^2 4x \, dx = \int 1 - \cos 8x \, dx$
$= x - \frac{1}{8}\sin 8x + c \; \#$

e) $\int \pi - \sin \pi x \, dx = \pi x + \frac{1}{\pi}\cos \pi x + c \; \#$

f) $\int 4\tan^2(2x+1) \, dx = 4\int \sec^2(2x+1) - 1 \, dx$
$= \dfrac{4\tan(2x+1)}{2} - 4x + c$
$= 2\tan(2x+1) - 4x + c \; \#$

g) $\int 3\sin^2(2x+1) \, dx = \int \frac{3}{2}(1 - \cos(4x+2)) \, dx$
$= \frac{3}{2}x - \frac{3}{8}\sin(4x+2) + c \; \#$

h) $\int (2 - \cos 3x)^2 \, dx = \int 4 - 4\cos 3x + \cos^2 3x \, dx$
$= 4x - \frac{4}{3}\sin 3x + \int \dfrac{\cos 6x + 1}{2} \, dx$
$= 4x - \frac{4}{3}\sin 3x + \frac{1}{2}x + \frac{1}{12}\sin 6x + c$
$= \frac{9}{2}x - \frac{4}{3}\sin 3x + \frac{1}{12}\sin 6x + c \; \#$

2a) $\int \sin 2x \cos 3x \, dx = \frac{1}{2}\int \sin 5x + \sin(-x) \, dx$
$= \frac{1}{2}\int \sin 5x - \sin x \, dx$
$= -\frac{1}{10}\cos 5x + \frac{1}{2}\cos x + c \; \#$

b) $\int \sin x \sin 3x \, dx = -\frac{1}{2}\int \cos 4x - \cos(-2x) \, dx$
$= -\frac{1}{2}\int \cos 4x - \cos 2x \, dx$
$= -\frac{1}{8}\sin 4x + \frac{1}{4}\sin 2x + c \; \#$

c) $\int \cos 4x \cos 5x \, dx = \frac{1}{2}\int \cos 9x + \cos(-x) \, dx$
$= \frac{1}{2}\int \cos 9x + \cos x \, dx$
$= \frac{1}{18}\sin 9x + \frac{1}{2}\sin x + c \; \#$

d) $\int \dfrac{\cos x}{\sqrt{\sin x}} \, dx = 2\sqrt{\sin x} + c \; \#$

e) $\int \dfrac{\sin^3 x}{\cos^2 x} \, dx = \int \sin x \tan^2 x \, dx$
$= \int \sin x (\sec^2 x - 1) \, dx$
$= \int \dfrac{\sin x}{\cos^2 x} - \sin x \, dx$
$= \dfrac{1}{\cos x} + \cos x + c \; \#$

3) $\sin 3x = \sin(2x+x)$

$= \sin 2x \cos x + \cos 2x \sin x$

$= 2\sin x \cos^2 x + (1-2\sin^2 x)\sin x$

$= 2\sin x(1-\sin^2 x) + \sin x - 2\sin^3 x$

$= 2\sin x - 2\sin^3 x + \sin x - 2\sin^3 x$

$= 3\sin x - 4\sin^3 x$ #

$\int \frac{\sin 3x}{\sin x}\,dx = \int \frac{3\sin x - 4\sin^3 x}{\sin x}\,dx$

$= \int 3 - 4\sin^2 x\,dx$

$= \int 3 - 2(1-\cos 2x)\,dx$

$= \int 1 + 2\cos 2x\,dx$

$= x + \sin 2x + c$ #

4) $\frac{d}{dx}\tan^3 x = 3\tan^2 x \cdot \sec^2 x$

$\int \left(\frac{\tan x}{\cos x}\right)^2 dx = \int \tan^2 x \cdot \sec^2 x\,dx$

$= \frac{1}{3}\tan^3 x + c$ #

5) $\int \sec^2 \frac{x}{2}\,dx = 2\tan\frac{x}{2} + c$

when $x = \frac{\pi}{3}$, $y = 5\sqrt{3}$:

$5\sqrt{3} = 2\tan\frac{\pi}{6} + c$

$c = 5\sqrt{3} - \frac{2}{\sqrt{3}}$

$= 5\sqrt{3} - \frac{2}{3}\sqrt{3}$

$= \frac{13\sqrt{3}}{3}$ # $\Rightarrow y = 2\tan\frac{x}{2} + \frac{13\sqrt{3}}{3}$ #

6) $\frac{d}{dx}\sin^3 2x = 3\sin^2 2x \cdot 2\cos 2x$

$= 6\sin^2 2x \cos 2x$

a) $\int_0^{\frac{\pi}{4}} \sin^2 2x \cos 2x\,dx = \left[\frac{1}{6}\sin^3 2x\right]_0^{\frac{\pi}{4}}$

$= \frac{1}{6}$ #

b) $\int_0^{\frac{\pi}{4}} \cos^3 2x\,dx = \int_0^{\frac{\pi}{4}} \cos 2x \cdot (1-\sin^2 2x)\,dx$

$= \int_0^{\frac{\pi}{4}} \cos 2x - \sin^2 2x \cos 2x\,dx$

$= \left[\frac{1}{2}\sin 2x - \frac{1}{6}\sin^3 2x\right]_0^{\frac{\pi}{4}}$

$= \left(\frac{1}{2} - \frac{1}{6}\right) - 0$

$= \frac{1}{3}$ #

7a) $y = \frac{2x}{\sin x} \Rightarrow \frac{dy}{dx} = \frac{2\sin x - 2x\cos x}{\sin^2 x}$

$= \frac{2}{\sin x} - \frac{2x\cos x}{\sin^2 x}$

b) $\int \frac{1}{\sin x} - \frac{x\cos x}{\sin^2 x}\,dx$

$= \frac{1}{2}\left(\frac{2x}{\sin x}\right) + c$

$= \frac{x}{\sin x} + c$

$\Rightarrow \left[\frac{x}{\sin x}\right]_{\frac{\pi}{2}}^{\frac{3\pi}{2}}$ cannot be evaluated because $\frac{1}{\sin x}$ is undefined for $x = \pi$.