

# Assignment 20 solutions

$$\begin{aligned}\cos 2x &= 1 - 2\sin^2 x \\ &= 2\cos^2 x - 1\end{aligned}$$

$$1a) \int 8\sec^2 5x \, dx = \frac{8 \tan 5x}{5} + C \quad \#$$

$$b) \int \cos \frac{x}{3} \, dx = 3 \sin \frac{x}{3} + C \quad \#$$

where C is a const.

$$c) \int x^2 + \sec^2 \frac{\pi}{2} x \, dx = \frac{x^3}{3} + \frac{2}{\pi} \tan \frac{\pi}{2} x + C \quad \#$$

$$\begin{aligned}d) \int 2 \sin^2 4x \, dx &= \int 1 - \cos 8x \, dx \\ &= x - \frac{1}{8} \sin 8x + C \quad \# \end{aligned}$$

$$e) \int \pi - \sin \pi x \, dx = \pi x + \frac{1}{\pi} \cos \pi x + C \quad \#$$

$$\begin{aligned}f) \int 4 + \tan^2(2x+1) \, dx &= 4 \int \sec^2(2x+1) - 1 \, dx \\ &= \frac{4 \tan(2x+1)}{2} - 4x + C \\ &= 2 \tan(2x+1) - 4x + C \quad \# \end{aligned}$$

$$\begin{aligned}g) \int 3 \sin^2(2x+1) \, dx &= \int \frac{3}{2} (1 - \cos(4x+2)) \, dx \\ &= \frac{3}{2} x - \frac{3}{8} \sin(4x+2) + C \quad \# \end{aligned}$$

$$\begin{aligned}h) \int (2 - \cos 3x)^2 \, dx &= \int 4 - 4 \cos 3x + \cos^2 3x \, dx \\ &= 4x - \frac{4}{3} \sin 3x + \int \frac{\cos 6x + 1}{2} \, dx \\ &= 4x - \frac{4}{3} \sin 3x + \frac{1}{2} x + \frac{1}{12} \sin 6x + C \\ &= \frac{9}{2} x - \frac{4}{3} \sin 3x + \frac{1}{12} \sin 6x + C \quad \# \end{aligned}$$

$$\begin{aligned}2a) \int \sin 2x \cos 3x \, dx &= \frac{1}{2} \int \sin 5x + \sin(-x) \, dx \\ &= \frac{1}{2} \int \sin 5x - \sin x \, dx \\ &= -\frac{1}{10} \cos 5x + \frac{1}{2} \cos x + C \quad \# \end{aligned}$$

$$\begin{aligned}b) \int \sin x \sin 3x \, dx &= -\frac{1}{2} \int \cos 4x - \cos(-2x) \, dx \\ &= -\frac{1}{2} \int \cos 4x - \cos 2x \, dx \\ &= -\frac{1}{8} \sin 4x + \frac{1}{4} \sin 2x + C \quad \# \end{aligned}$$

$$\begin{aligned}c) \int \cos 4x \cos 5x \, dx &= \frac{1}{2} \int \cos 9x + \cos(-x) \, dx \\ &= \frac{1}{2} \int \cos 9x + \cos x \, dx \\ &= \frac{1}{18} \sin 9x + \frac{1}{2} \sin x + C \quad \# \end{aligned}$$

$$d) \int \frac{\cos x}{\sqrt{\sin x}} \, dx = 2\sqrt{\sin x} + C \quad \#$$

$$\begin{aligned}e) \int \frac{\sin^3 x}{\cos^2 x} \, dx &= \int \sin x \tan^2 x \, dx \\ &= \int \sin x (\sec^2 x - 1) \, dx \\ &= \int \frac{\sin x}{\cos^2 x} - \sin x \, dx \\ &= \frac{1}{\cos x} + \cos x + C \quad \# \end{aligned}$$

$$\begin{aligned}
 3) \quad \sin 3x &= \sin(2x+x) \\
 &= \sin 2x \cos x + \cos 2x \sin x \\
 &= 2 \sin x \cos^2 x + (1-2\sin^2 x) \sin x \\
 &= 2 \sin x (1-\sin^2 x) + \sin x - 2 \sin^3 x \\
 &= 2 \sin x - 2 \sin^3 x + \sin x - 2 \sin^3 x \\
 &= 3 \sin x - 4 \sin^3 x \quad \#
 \end{aligned}$$

$$\begin{aligned}
 \int \frac{\sin 3x}{\sin x} dx &= \int \frac{3 \sin x - 4 \sin^3 x}{\sin x} dx \\
 &= \int 3 - 4 \sin^2 x dx \\
 &= \int 3 - 2(1 - \cos 2x) dx \\
 &= \int 1 + 2 \cos 2x dx \\
 &= x + \sin 2x + C \quad \#
 \end{aligned}$$

$$\begin{aligned}
 4) \quad \frac{d}{dx} \tan^3 x &= 3 \tan^2 x \cdot \sec^2 x \\
 \int \left( \frac{\tan x}{\cos x} \right)^2 dx &= \int \tan^2 x \cdot \sec^2 x dx \\
 &= \frac{1}{3} \tan^3 x + C \quad \#
 \end{aligned}$$

$$5) \int \sec^2 \frac{x}{2} dx = 2 \tan \frac{x}{2} + C$$

when  $x = \frac{\pi}{3}$ ,  $y = 5\sqrt{3}$ :

$$5\sqrt{3} = 2 \tan \frac{\pi}{6} + C$$

$$C = 5\sqrt{3} - \frac{2}{\sqrt{3}}$$

$$= 5\sqrt{3} - \frac{2}{\sqrt{3}}$$

$$= \frac{13\sqrt{3}}{3} \quad \# \Rightarrow y = 2 \tan \frac{x}{2} + \frac{13\sqrt{3}}{3} \quad \#$$

$$\begin{aligned}
 7a) \quad y = \frac{2x}{\sin x} &\Rightarrow \frac{dy}{dx} = \frac{2 \sin x - 2x \cos x}{\sin^2 x} \\
 &= \frac{2}{\sin x} - \frac{2x \cos x}{\sin^2 x}
 \end{aligned}$$

$$b) \int \frac{1}{\sin x} - \frac{x \cos x}{\sin^2 x} dx$$

$$= \frac{1}{2} \left( \frac{2x}{\sin x} \right) + C$$

$$= \frac{x}{\sin x} + C$$

$\Rightarrow \left[ \frac{x}{\sin x} \right]_{\frac{\pi}{2}}^{\frac{3\pi}{2}}$  cannot be evaluated because  $\frac{1}{\sin x}$  is undefined for  $x = \pi$ .

$$\begin{aligned}
 6) \quad \frac{d}{dx} \sin^3 2x &= 3 \sin^2 2x \cdot 2 \cos 2x \\
 &= 6 \sin^2 2x \cos 2x
 \end{aligned}$$

$$\begin{aligned}
 a) \quad \int_0^{\frac{\pi}{4}} \sin^2 2x \cos 2x dx &= \left[ \frac{1}{6} \sin^3 2x \right]_0^{\frac{\pi}{4}} \\
 &= \frac{1}{6} \quad \#
 \end{aligned}$$

$$\begin{aligned}
 b) \quad \int_0^{\frac{\pi}{4}} \cos^3 2x dx &= \int_0^{\frac{\pi}{4}} \cos 2x \cdot (1 - \sin^2 2x) dx \\
 &= \int_0^{\frac{\pi}{4}} \cos 2x - \sin^2 2x \cos 2x dx \\
 &= \left[ \frac{1}{2} \sin 2x - \frac{1}{6} \sin^3 2x \right]_0^{\frac{\pi}{4}} \\
 &= \left( \frac{1}{2} - \frac{1}{6} \right) - 0 \\
 &= \frac{1}{3} \quad \#
 \end{aligned}$$

