Year 4 Math Assignment 19: Integration

1. Integrate the following with respect to x:

   (a) $(3x+5)^3$ (b) $\sqrt{(7-2x)^5}$

   (c) $\dfrac{3}{(2x+9)^2}$ (d) $x(13-2x^2)$

   (e) $\dfrac{6x^2}{\sqrt{10+3x^3}}$ (f) $\sqrt{x}(1-\sqrt{x})$

2. Evaluate the following:

   (a) $\int_0^1 4\sqrt{3x+1}\,\mathrm{d}x$ (b) $\int_0^1 \left(\dfrac{3}{2x+1}\right)^2 \mathrm{d}x$

   (c) $\int_{-1}^1 \dfrac{2x^3+4x^2}{x^2}\,\mathrm{d}x$

3. Show that $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{2x}{\sqrt{x+1}}\right)=\dfrac{x+2}{(x+1)^{3/2}}$. Hence, evaluate $\int_0^8 \dfrac{x+2}{(x+1)^{3/2}}\,\mathrm{d}x$.

4. Given that $y=x\sqrt{2x^2+1}$, show that $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{1+4x^2}{\sqrt{1+2x^2}}$.

   Hence, evaluate $\int_0^2 \dfrac{3+12x^2}{\sqrt{1+2x^2}}\,\mathrm{d}x$.

5. Find the possible values of p if $\int_3^p (5-x)^5\,\mathrm{d}x=\dfrac{21}{2}$.

6. Given that $\int_1^3 \mathrm{h}(x)\,\mathrm{d}x=5$, evaluate

   (i) $\int_1^3 [\mathrm{h}(x)+x]\,\mathrm{d}x$ (ii) $\int_3^1 [\mathrm{h}(x)-x]\,\mathrm{d}x$

   Find the value of k for which $\int_1^3 \left[kx^2+\mathrm{h}(x)\right]\,\mathrm{d}x=31$.

7. By expressing as partial fractions or otherwise, integrate the following:

   (a) $\dfrac{4x^2-4x+9}{(2x-1)^2}$ (b) $\dfrac{12x^2+36x+1}{(2x+3)^2}$

   (c) $\dfrac{3x^3+27x^2+81x+21}{(x+3)^3}$ (d) $\dfrac{-x^4+2x^3-6x^2+2x-1}{(x^2+1)^2(1-x)^2}$