Assignment 19 solutions

1a) $\int(3x+5)^3dx = \frac{(3x+5)^4}{12}+C$ #

b) $\int(7-2x)^{\frac{5}{2}}dx = -\frac{(7-2x)^{\frac{7}{2}}}{7}+C$ # where C is a const.

c) $\int 3(2x+9)^{-2}dx = \frac{3(2x+9)^{-1}}{-2}+C$

$= -\frac{3}{2(2x+9)}+C$ #

d) $\int x(13-2x^2)dx = \int 13x-2x^3dx$

$= \frac{13x^2}{2}-\frac{x^4}{2}+C$ #

e) $\int \frac{6x^2}{\sqrt{10+3x^3}}dx = \frac{2}{3}\int\frac{9x^2}{\sqrt{10+3x^3}}dx$

$= \frac{4}{3}(10+3x^3)^{\frac{1}{2}}+C$ #

f) $\int\sqrt{x}(1-\sqrt{x})dx = \int x^{\frac{1}{2}}-x\ dx$

$= \frac{2x^{\frac{3}{2}}}{3}-\frac{x^2}{2}+C$ #

2a) $\int_0^1 4\sqrt{3x+1}\,dx = 8\frac{(3x+1)^{\frac{3}{2}}}{9}\Big|_0^1$

$= \frac{64}{9}-\frac{8}{9} = \frac{56}{9}$ #

b) $\int_0^1\left(\frac{3}{2x+1}\right)^2dx = 9\int_0^1(2x+1)^{-2}dx$

$= \left[\frac{9(2x+1)^{-1}}{-2}\right]_0^1$

$= -\frac{3}{2}+\frac{9}{2}$

$= 3$ #

c) $\int_{-1}^1\frac{2x^3+4x^2}{x^2}dx = \int_{-1}^1 2x+4\ dx$

$= [x^2+4x]_{-1}^1$

$= 5-(-3)$

$= 8$ #

3) $\frac{d}{dx}\frac{2x}{\sqrt{x+1}} = \frac{2\sqrt{x+1}-2x(\frac{1}{2})(x+1)^{-\frac{1}{2}}}{(x+1)}$

$= \frac{2\sqrt{x+1}-\frac{x}{\sqrt{x+1}}}{x+1}$

$= \frac{2(x+1)-x}{(x+1)^{\frac{3}{2}}}$

$= \frac{x+2}{(x+1)^{3/2}}$ # shown

$\int_0^8\frac{x+2}{(x+1)^{3/2}}dx = \left[\frac{2x}{\sqrt{x+1}}\right]_0^8$

$= \frac{16}{3}$ #

4) $y = x\sqrt{2x^2+1}$

$\frac{dy}{dx} = x(\frac{1}{2})(2x^2+1)^{-\frac{1}{2}}(4x)+\sqrt{2x^2+1}$

$= (2x^2+1)^{-\frac{1}{2}}[2x^2+2x^2+1]$

$= \frac{1+4x^2}{\sqrt{1+2x^2}}$ # shown

$\int_0^2\frac{3+12x^2}{\sqrt{1+2x^2}}dx = 3\int_0^2\frac{1+4x^2}{\sqrt{1+2x^2}}dx$

$= 3\left[x\sqrt{2x^2+1}\right]_0^2$

$= 18$ #

5) $\int_3^p(5-x)^5dx = \left[\frac{(5-x)^6}{-6}\right]_3^p$

$\frac{21}{2} = \frac{(5-p)^6}{-6}-(-\frac{32}{3})$

$(5-p)^6 = 1$

$p = \pm1+5$

$= 6$ or 4 #

6i) $\int_1^3 h(x)+x\,dx = \int_1^3 h(x)dx + \frac{x^2}{2}\Big|_1^3$

$= 5 + \frac{9}{2} - \frac{1}{2}$

$= 9$ #

ii) $\int_3^1 h(x) - x\,dx = -\int_1^3 h(x) - x\,dx$

$= -\int_1^3 h(x)\,dx + \int_1^3 x\,dx$

$= -5 + 4$

$= -1$ #

$\int_1^3 kx^2 + h(x)\,dx = 31$

$\frac{kx^3}{3}\Big|_1^3 + 5 = 31$

$9k - \frac{1}{3}k = 26$

$k = 3$ #

7a) $\frac{4x^2-4x+9}{(2x-1)^2} = \frac{(2x-1)^2+8}{(2x-1)^2}$

$= 1 + \frac{8}{(2x-1)^2}$

$\therefore \int 1 + \frac{8}{(2x-1)^2}\,dx = x + \frac{8(2x-1)^{-1}}{-2}$

$= x - \frac{4}{2x-1} + C$ #

b) $\int \frac{12x^2+36x+1}{(2x+3)^2}\,dx = \int \frac{3(2x+3)^2 - 26}{(2x+3)^2}\,dx$

$= \int 3 - \frac{26}{(2x+3)^2}\,dx$

$= 3x + \frac{13}{-(2x+3)} + C$ #

c) $\frac{3x^3+27x^2+81x+21}{(x+3)^3} = \frac{3(x+3)^3 - 60}{(x+3)^3}$

$= 3 - \frac{60}{(x+3)^3}$

$\int 3 - \frac{60}{(x+3)^3}\,dx = 3x + \frac{30}{(x+3)^2} + C$ #

7d) $\frac{-x^4+2x^3-6x^2+2x-1}{(x^2+1)^2(1-x)^2}$

$= \frac{Ax+B}{x^2+1} + \frac{Cx+D}{(x^2+1)^2} + \frac{E}{1-x} + \frac{F}{(1-x)^2}$

By cover-up rule,

let $x=1$, $F=-1$

$-x^4+2x^3-6x^2+2x-1 = (Ax+B)(x^2+1)(1-x)^2 + (Cx+D)(1-x)^2 + E(1-x)(x^2+1)^2 - (x^2+1)^2$

let $x=0$, $-1 = B+D+E-1$

$B+D+E = 0$

$-x^4+2x^3-6x^2+2x-1 = (Ax+B)\overbrace{(x^2+1)(x^2-2x+1)}^{x^4-2x^3+x^2-2x+1} + ((Cx+D)(x^2-2x+1) + E(1-x)(x^4+2x^2+1) - (x^4+2x^2+1)$

compare coeff of

x^5: $0 = A - E \Rightarrow A = E$

x^4: $-1 = B - 2A + E - 1$ — (1)

x^3: $2 = A - 2B + C - 2E$ — (2)

(4) — x^2: $-6 = -2A+B+D-2C+2E-2$

x: $2 = -2B + A + C - 2D - E$ — (3)

Fr(1): $B - A = 0 \Rightarrow B = A$

Fr(2): $2 = -3A + C$ — (5)

Fr(3): $2 = -2A + C - 2D$ — (6)

Fr(4): $-4 = A + D - 2C$ — (7)

Fr(5), $C = 2 + 3A$

$\Rightarrow$(6): $2 = -2A + 2 + 3A - 2D$

$A = 2D$

$\Rightarrow$(7): $-4 = 2D + D - 2(2 + 3(2D))$

$-4 = 3D - 4 - 12D$

$9D = 0$ $\therefore A=0, B=0,$

$D = 0$ $\quad C=2, E=0$

7d) cont'd:

$$\frac{2x}{(x^2+1)^2} - \frac{1}{(1-x)^2} \#$$

$$\int \frac{2x}{(x^2+1)^2} - \frac{1}{(1-x)^2}\, dx$$

$$= -\frac{1}{(x^2+1)} - \frac{1}{(1-x)} + C \#$$

AH 7d)

$$\frac{-x^4+2x^3-6x^2+2x-1}{(x^2+1)^2(1-x)^2}$$

$$= \frac{-(x^2+1)^2+2x^3-4x^2+2x}{(x^2+1)^2(1-x)^2}$$

$$= -\frac{1}{(1-x)^2} + \frac{2x(1-x)^2}{(x^2+1)^2(1-x)^2}$$

$$= -\frac{1}{(1-x)^2} + \frac{2x}{(x^2+1)^2} \#$$

$$\therefore \int -\frac{1}{(1-x)^2} + \frac{2x}{(x^2+1)^2}\, dx$$

$$= -\frac{1}{1-x} - \frac{1}{x^2+1} + c \#$$

where c is a const.