Assignment 18 solutions

1a) $\int \frac{1}{\sqrt[3]{x}}\,dx = \frac{3x^{\frac{2}{3}}}{2} + C$ #

b) $\int \sqrt{x}+2\,dx = \frac{2x^{\frac{3}{2}}}{3} + 2x + C$ #

c) $\int 2x^3 - \frac{3}{x^2}\,dx = \frac{2x^4}{4} - \frac{3x^{-1}}{-1} + C$

$= \frac{1}{2}x^4 + \frac{3}{x} + C$ #

d) $\int 2x^2 - 3x\,dx = \frac{2x^3}{3} - \frac{3x^2}{2} + C$ #

e) $\int \frac{x^2+6x+9}{\sqrt{x}}\,dx = \int x^{\frac{3}{2}} + 6x^{\frac{1}{2}} + 9x^{-\frac{1}{2}}\,dx$

$= \frac{2x^{\frac{5}{2}}}{5} + 4x^{\frac{3}{2}} + 18x^{\frac{1}{2}} + C$ #

where C is a const.

f) $\int \frac{x^2}{4} + \frac{x}{2} - \frac{3}{4}\,dx = \frac{x^3}{12} + \frac{x^2}{4} - \frac{3}{4}x + C$ #

2i) $\int 3x + k\,dx = \frac{3x^2}{2} + kx + C$ where C is a const

when $x = 2$, $\frac{dy}{dx} = 0 \Rightarrow 6 + k = 0$

$k = -6$ #

(ii) $y = \frac{3x^2}{2} - 6x + C$

At $(2,5)$,

$5 = 6 - 12 + C$

$C = 11$

$\Rightarrow y = \frac{3x^2}{2} - 6x + 11$ #

3) $\int 3t^2 - 7\,dt = \frac{3t^3}{3} - 7t + C$ where C is a const.

$S = t^3 - 7t + C$

$t = 0$, $S = 6 \Rightarrow 6 = C$

$\therefore S = t^3 - 7t + 6$ #

4i) $V = -\frac{0.012}{3/2} t^{\frac{3}{2}} + 15$

$= -0.008\,t^{\frac{3}{2}} + 15$ #

(ii) when $v = 0$, $15 = 0.008\,t^{\frac{3}{2}}$

$t^{\frac{3}{2}} = 1875$

$t = 152.055\ldots$

≈ 152 mins #

Q5) $\int 6x - 4\,dx = \frac{6x^2}{2} - 4x + C$ where C is a const

$x = 2$, $\frac{dy}{dx} = 10 \Rightarrow 10 = 3(4) - 8 + C$

$C = 6$

$\Rightarrow \frac{dy}{dx} = 3x^2 - 4x + 6$

$\int 3x^2 - 4x + 6\,dx = \frac{3x^3}{3} - \frac{4x^2}{2} + 6x + C_1$ where C_1 is a const

$y = x^3 - 2x^2 + 6x + C_1$

At $(2, 5)$,

$5 = 8 - 8 + 6(2) + C_1$

$C_1 = -7$

$\therefore$ eq^n: $y = x^3 - 2x^2 + 6x - 7$ ☆

Q6) $\frac{dy}{dx} = K(x-4)(x+2)$, $K = 2$ (from further info given).

$\int 2x^2 - 4x - 16\,dx = \frac{2x^3}{3} - 2x^2 - 16x + C$ where C is a const.

At $(3, 0)$,

$0 = 18 - 18 - 48 + C$

$C = 48$

$\therefore$ eq^n: $y = \frac{2}{3}x^3 - 2x^2 - 16x + 48$ ☆

Q7) $\int_3^2 f(x)\,dx + \int_2^5 f(x)\,dx$

$= -\int_2^3 f(x)\,dx + \int_2^5 f(x)\,dx$

$= -4 + (4 + 7)$

$= 7$ ☆