

Year 4 Math Assignment 15: Differentiation Solutions

1. (a) $y = 5^x \cos 2x$
 $\frac{dy}{dx} = 5^x(-2 \sin 2x) + \cos 2x(5^x \ln 5)$

(b) $y = \frac{3^x}{x^2}$
 $\frac{dy}{dx} = 3^x(-2x^{-3}) + x^{-2}(3^x \ln 3)$

(c) $y = 3^x \ln 5x$
 $\frac{dy}{dx} = 3^x\left(\frac{1}{x}\right) + (\ln 5x)(3^x \ln 3)$
 $= \frac{3^x}{x} + 3^x \ln 3 \ln 5x$

2. (a) $\frac{dy}{dx} = 3e^{9x}$

(b) $\frac{dy}{dx} = -2(e^{1-x})^2$

(c) $\frac{dy}{dx} = e^{-x}(-2 \sin 2x) + \cos 2x(-e^{-x})$
 $= -2e^{-x} \sin 2x - e^{-x} \cos 2x$

(d) $\frac{dy}{dx} = \frac{(1+e^{2x})(e^x) - e^x(2e^{2x})}{(1+e^{2x})^2}$
 $= \frac{e^x - e^{3x}}{(1+e^{2x})^2}$

3. (a) $\ln[(x+1)(x-3)] = \ln(x+1) + \ln(x-3)$
 $\frac{dy}{dx} = \frac{1}{x+1} + \frac{1}{x-3}$

(b) $\frac{dy}{dx} = \frac{\cos x - \sin x}{\sin x + \cos x}$

(c) $\frac{dy}{dx} = \frac{e^x}{1+e^x}$

(d) $\frac{dy}{dx} = \frac{1}{\sqrt{x^2+1-x}} \left(\frac{1}{2}(x^2+1)^{-\frac{1}{2}}(2x) - 1 \right)$
 $= \frac{1}{\sqrt{x^2+1-x}} \left((x^2+1)^{-\frac{1}{2}}(x) - 1 \right)$

4. (a) $\frac{dy}{dx} = (3 + \cos x)e^{3x + \sin x}$

(b) $\frac{dy}{dx} = \frac{1}{2\sqrt{x}(1+\sqrt{x})}$

(c) $\frac{dy}{dx} = \frac{(x+1)2e^{2x} - e^{2x}}{(x+1)^2}$
 $= \frac{e^{2x}(2x+1)}{(x+1)^2}$

(d) $y = \frac{1}{3}[\ln(x+2) - \ln(2-x)]$
 $\frac{dy}{dx} = \frac{1}{3} \left[\frac{1}{x+2} - \frac{1}{2-x} \right]$

5.

$$\ln(x^2 - 3) = 0$$

$$x^2 - 3 = 1$$

$$x = 2 \text{ or } -2 \text{ (rej)}$$

Coordinate is (2, 0)

$$\frac{dy}{dx} = \frac{2x}{x^2 - 3}$$

$$\left. \frac{dy}{dx} \right|_{x=2} = 4$$

$$\text{gradient of normal} = -\frac{1}{4}$$

$$\text{equation of normal} \rightarrow y = -\frac{1}{4}x + \frac{1}{2}$$

6.

$$e^{\sqrt{x}-2} = 1$$

$$\sqrt{x} - 2 = 0$$

$$\sqrt{x} = 2$$

$$x = 4 \rightarrow \text{Coordinate is } (4, 1)$$

$$\frac{dy}{dx} = \left(\frac{1}{2\sqrt{x}} \right) e^{\sqrt{x}-2}$$

$$\left. \frac{dy}{dx} \right|_{x=4} = \frac{1}{4}$$

$$\text{equation of tangent} \rightarrow y - 1 = \frac{1}{4}(x - 4)$$

$$y = \frac{1}{4}x \rightarrow \text{Tangent passes through origin}$$

7.

$$y = \ln(3x^2 - 2)$$

$$\frac{dy}{dx} = \frac{6x}{3x^2 - 2}$$

$$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$$

$$= \frac{6x}{3x^2 - 2} \times 3$$

$$\left. \frac{dy}{dt} \right|_{x=1} = 18 \text{ units/s}$$