Assignment 14 solutions.

Q1(a) $y' = 2\sec^2(1-x^3).(-3x^2)$

$= -6x^2\sec^2(1-x^3)$ #

(b) $y' = 2\cos\left(\frac{1}{x}\right).\left(-\frac{1}{x^2}\right)$

$= -\frac{2}{x^2}\cos\left(\frac{1}{x}\right)$ #

(c) $y' = -5(-\sin(x-2x^3))(1-6x^2)$

$= 5(1-6x^2)\sin(x-2x^3)$ #.

(d) $y' = 2(1+\sin x)(\cos x)$ #

(e) $y' = \frac{1}{2}[\cos x]^{-\frac{1}{2}}(-\sin x)$

$= -\frac{\sin x}{2\sqrt{\cos x}}$ #

(f) $y' = 4\tan^3 2x(\sec^2 2x)(2) - 3\cos^2 5x(-\sin 5x)(5)$

$= 8\tan^3 2x\sec^2 2x + 15\cos^2 5x\sin 5x$

$= 8\sin 2x + 15\cos^2 5x\sin 5x$ #

Q2) $y' = \cos x - 3\sin x$

LHS $\Rightarrow \cos x(\cos x - 3\sin x) + y\sin x$

$= \cos^2 x - 3\sin x\cos x + (\sin x + 3\cos x)\sin x$

$= \cos^2 x - 3\sin x\cos x + \sin^2 x + 3\sin x\cos x$

$= 1$ # shown

Q3) $y' = 3A\cos 3x - 3B\sin 3x$

$y'' = -9A\sin 3x - 9B\cos 3x$

LHS $\Rightarrow -9A\sin 3x - 9B\cos 3x + 9A\sin 3x + 9B\cos 3x = 0$ # shown.

Q4) $y = x + \sin 3x$

$y' = 1 + 3\cos 3x$

$x = \frac{\pi}{3}, y = \frac{\pi}{3}, y' = -2$

Eqn of tangent: $y - \frac{\pi}{3} = -2\left(x - \frac{\pi}{3}\right)$

$y = -2x + \pi$ #

Eqn of normal: $y - \frac{\pi}{3} = \frac{1}{2}\left(x - \frac{\pi}{3}\right)$

$y = \frac{1}{2}x + \frac{\pi}{6}$ #

Q5) $AB + BC + CD$

$= 7\cos x + (7\sin x - \cos x) + \sin x$

$= 8\sin x + 6\cos x$ # shown.

$8\cos x - 6\sin x = 0$

$8\cos x = 6\sin x$

$\tan x = \frac{4}{3}$

$x = 0.927$ #

$-8\sin x - 6\cos x\big|_{x=0.927} = -10 < 0$

$\Rightarrow$ max value

Max value $= 10$ #