

Assignment 14 solutions.

$$\begin{aligned} \text{Q1(a)} \quad y' &= 2 \sec^2(1-x^3) \cdot (-3x^2) \\ &= -6x^2 \sec^2(1-x^3) \# \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad y' &= 2 \cos\left(\frac{1}{x}\right) \cdot \left(-\frac{1}{x^2}\right) \\ &= -\frac{2}{x^2} \cos\left(\frac{1}{x}\right) \# \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad y' &= -5(-\sin(x-2x^3))(1-6x^2) \\ &= 5(1-6x^2)\sin(x-2x^3) \# \end{aligned}$$

$$\text{(d)} \quad y' = 2(1+\sin x)(\cos x) \#$$

$$\begin{aligned} \text{(e)} \quad y' &= \frac{1}{2}[\cos x]^{-\frac{1}{2}}(-\sin x) \\ &= -\frac{\sin x}{2\sqrt{\cos x}} \# \end{aligned}$$

$$\begin{aligned} \text{(f)} \quad y' &= 4 \tan^3 2x (\sec^2 2x) (2) - 3 \cos^2 5x (-\sin 5x) (5) \\ &= 8 \tan^3 2x \sec^2 2x + 15 \cos^2 5x \sin 5x \\ &= 8 \sin 2x + 15 \cos^2 5x \sin 5x \# \end{aligned}$$

$$\text{Q2)} \quad y' = \cos x - 3 \sin x$$

$$\begin{aligned} \text{LHS} &\Rightarrow \cos x (\cos x - 3 \sin x) + y \sin x \\ &= \cos^2 x - 3 \sin x \cos x + (\sin x + 3 \cos x) \sin x \\ &= \cos^2 x - 3 \sin x \cos x + \sin^2 x + 3 \sin x \cos x \\ &= 1 \# \text{ shown} \end{aligned}$$

$$\text{Q3)} \quad y' = 3A \cos 3x - 3B \sin 3x$$

$$y'' = -9A \sin 3x - 9B \cos 3x$$

$$\text{LHS} \Rightarrow -9A \sin 3x - 9B \cos 3x + 9A \sin 3x + 9B \cos 3x = 0 \# \text{ shown}$$

$$\begin{aligned} \text{Q4)} \quad y &= x + \sin 3x \\ y' &= 1 + 3 \cos 3x \end{aligned}$$

$$x = \frac{\pi}{3}, y = \frac{\pi}{3}, y' = -2$$

$$\begin{aligned} \text{Eq}^n \text{ of tangent: } \quad y - \frac{\pi}{3} &= -2(x - \frac{\pi}{3}) \\ y &= -2x + \pi \# \end{aligned}$$

$$\begin{aligned} \text{Eq}^n \text{ of normal: } \quad y - \frac{\pi}{3} &= \frac{1}{2}(x - \frac{\pi}{3}) \\ y &= \frac{1}{2}x + \frac{\pi}{6} \# \end{aligned}$$

$$\text{Q5)} \quad AB + BC + CD$$

$$= 7 \cos x + (7 \sin x - \cos x) + \sin x$$

$$= 8 \sin x + 6 \cos x \# \text{ shown}$$

$$8 \cos x - 6 \sin x = 0$$

$$8 \cos x = 6 \sin x$$

$$\tan x = \frac{4}{3}$$

$$x = 0.927 \#$$

$$-8 \sin x - 6 \cos x \Big|_{x=0.927} = -10 < 0$$

\Rightarrow max value

$$\text{Max value} = 10 \#$$