Year 4 Math Assignment 13: Rate of Change Solutions

Q1 (i)

$f'(x) = 2x + b$

$5 = 8 + b \rightarrow b = -3$

$0 = 16 + 4b + c \rightarrow c = -4$

(ii)

$f'(x) = 2x - 3$

For $2 < x < 6$,

$f'(x) > 0 \rightarrow y = \mathrm{f}(x)$ is an increasing function

(iii)

$f'(x) = 2x - 3$

$x = 2,\ f'(x) = 1$

$x = 6,\ f'(x) = 9$

$\therefore 1 < f'(x) < 9$

Q2 $\dfrac{dV}{dx} = 0.03x^2 + 4.4x + 200$

(i) $\dfrac{dx}{dt} = \dfrac{dx}{dV} \times \dfrac{dV}{dt}$

$$= \frac{1}{0.03(10)^2 + 4.4(10) + 200} \times 60 = \frac{60}{247}$$

The depth is increasing at the rate of 0.243 cm s^{-1}

(ii) $\dfrac{dx}{dV} \times \dfrac{dV}{dt} = 0.2$

$$\frac{1}{0.03x^2 + 4.4x + 200} \times 60 = 0.2$$

$$0.006x^2 + 0.88x - 20 = 0$$

$$x = \frac{-0.88 \pm \sqrt{0.88^2 - 4(0.006)(-20)}}{2(0.006)} = 20$$

The depth of water is 20 cm

Q3

(i) T*he phrase "**water is poured at a steady rate**" implies a uniform rate of change,*

i.e. $\dfrac{dV}{dt}$ *is a constant.* Hence it can be obtained by

$$\frac{dV}{dt} = \frac{total\ volume\ of\ water\ transferred}{total\ time\ taken}$$

$$= \frac{\pi(3)^2(5)}{3} = 15\pi$$

(ii) $\dfrac{dx}{dt} = \dfrac{dx}{dV} \times \dfrac{dV}{dt} = \dfrac{1}{\pi x^2} \times 15\pi$

$$\left.\frac{dx}{dt}\right|_{x=2.5} = 2.4$$

x is increasing at the rate of 2.4 cm s^{-1}.

Q4 *Using similar triangles,*

$$\frac{r}{x} = \frac{20}{60} \Rightarrow r = \frac{x}{3}$$

$$V = \frac{1}{3}\pi r^2 x = \frac{1}{3}\pi\left(\frac{x}{3}\right)^2 x = \frac{\pi x^3}{27}(shown)$$

$$\frac{dV}{dx} = \frac{\pi x^2}{9}$$

$$\frac{dx}{dt} = \frac{dx}{dV} \times \frac{dV}{dt} = \frac{9}{\pi x^2} \times 40 = \frac{360}{\pi x^2}$$

Rate of increase of the depth of the water is $\dfrac{360}{\pi x^2} cms^{-1}$.

(ii) $A = \pi r^2 = \pi\left(\dfrac{x}{3}\right)^2 \Rightarrow \dfrac{dA}{dx} = \dfrac{2\pi x}{9}$

$$\frac{dA}{dt} = \frac{dA}{dx} \times \frac{dx}{dt} = \frac{2\pi(2)}{9} \times \frac{90}{\pi} = 40$$

Rate of increase of the area of the horizontal surface of the water is $40\ cm^2 s^{-1}$.

Q5

Sub $y = p$ into $y^2 = 4x$

Coordinate of Q: $\left(\dfrac{p^2}{4}, p\right)$

Area of PQR, $A = \dfrac{1}{2} \times p \times \left(\dfrac{p^2}{4} - 2\right) = \dfrac{p^3}{8} - p$

$$\frac{dA}{dt} = \frac{dA}{dp} \times \frac{dp}{dt}$$

$$= \left(\frac{3p^2}{8} - 1\right) \times 0.2$$

$$\left.\frac{dA}{dt}\right|_{p=6} = \left(\frac{3(6)^2}{8} - 1\right) \times 0.2$$

$= 2.5$ units per sec