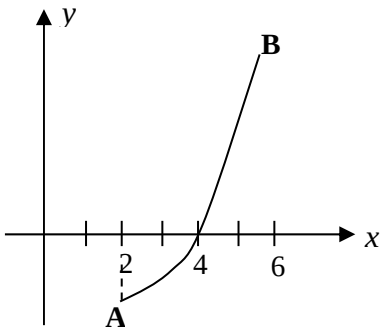
Year 4 Math Assignment 13: Differentiation **Time: 60 mins**

1 In the diagram, AB is a curve which represents the function $y = \mathrm{f}(x)$ where $\mathrm{f}(x) = x^2 + bx + c$ for $2 < x < 6$. The x-coordinates of A and B are 2 and 6 respectively. AB cuts the x-axis at that point where $x = 4$ and $\mathrm{f}'(4) = 5$.



(i) Find the values of b and c.
(ii) Show that $y = \mathrm{f}(x)$ is an increasing function.
(iii) Find the range of values of f'(x) for $2 < x < 6$.

2 Liquid is poured into a bucket at a rate of $60\,\mathrm{cm^3 s^{-1}}$. The volume, $V\ cm^3$, of liquid in the bucket, when the depth of liquid is x cm, is given by $V = 0.01x^3 + 2.2x^2 + 200x$. Find
(i) the rate of increase in the depth of liquid when $x = 10$,
(ii) the depth of liquid when the rate of increase in the depth is $0.2\,\mathrm{cms^{-1}}$.

3 A cylindrical jar, of radius 3 cm, contains water to a depth of 5 cm. The water is then poured at a steady rate into an inverted conical container with its axis vertical. After t seconds the depth of water in this container is x cm and the volume, V ml, of water that has been transferred is given by $V = \frac{1}{3}\pi x^3$.

Given that all the water is transferred in 3 seconds, find
(i) $\frac{\mathrm{d}V}{\mathrm{d}t}$ in terms of π, (ii) the rate at which x is increasing at the moment when x = 2.5.

4 A container is in the shape of an inverted right circular cone of height 60 cm and base radius 20 cm. Water is poured into the container at a constant rate of $40\,\mathrm{cm^3 s^{-1}}$.

(a) Given that the depth of water in the container is x cm, find an expression, in terms of x, for the volume of water in the container.
(b) Calculate the rate of increase of
(i) the depth of the water,
(ii) the area of the horizontal surface of the water at the instant when the depth of water is 2 cm.

5 The diagram shows part of the curve $y^2 = 4x$. The point P is on the x-axis and the point Q is on the curve; PQ is parallel to the y-axis and is p units in length. Given that R is the point (2, 0), express the area, A, of the triangle PQR in terms of p. The point P moves along the x-axis and the point Q moves along the curve in such a way that PQ remains parallel to the y-axis and p increases at the rate of 0.2 units per second. Find the rate at which A is increasing at the instant when $p = 6$ units