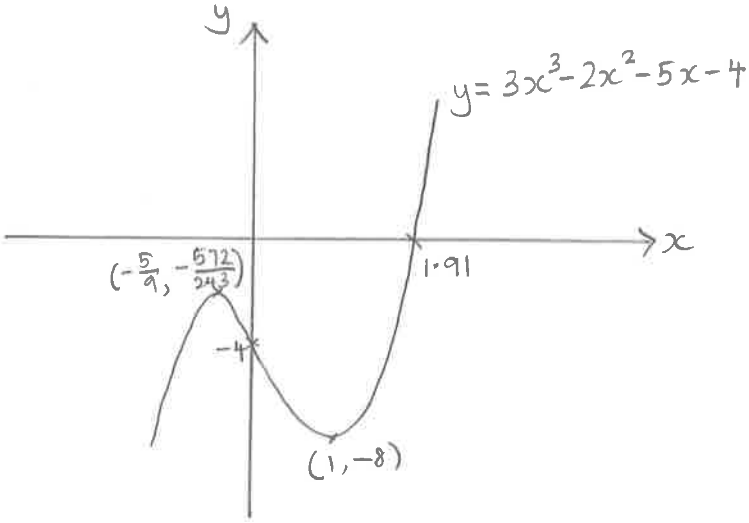
Assignment 12 sol^{ns}

Q1(a) $y = 3x^3 - 2x^2 - 5x - 4$



$x = 0,\ y = -4 \qquad (0,-4)$

$y = 0,\ x = 1.91 \qquad (1.91, 0)$

$y' = 9x^2 - 4x - 5 = 0$

$x = 1$ or $-\frac{5}{9}$

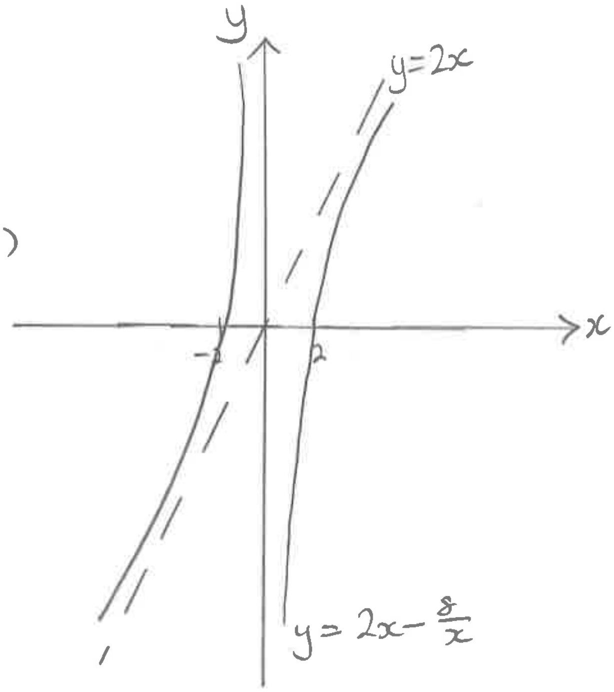
$y'' = 18x - 4$

$\Rightarrow (1,-8)$ is min pt

$(-\frac{5}{9}, -\frac{572}{243})$ is max pt

$\to (-0.556, -2.35)$

(b) $y = 2x - \frac{8}{x}$



$x = 0$ is vertical asymptote

$y = 0,\ 2x^2 - 8 = 0$

$x = \pm 2 \qquad (2,0)$ and $(-2,0)$

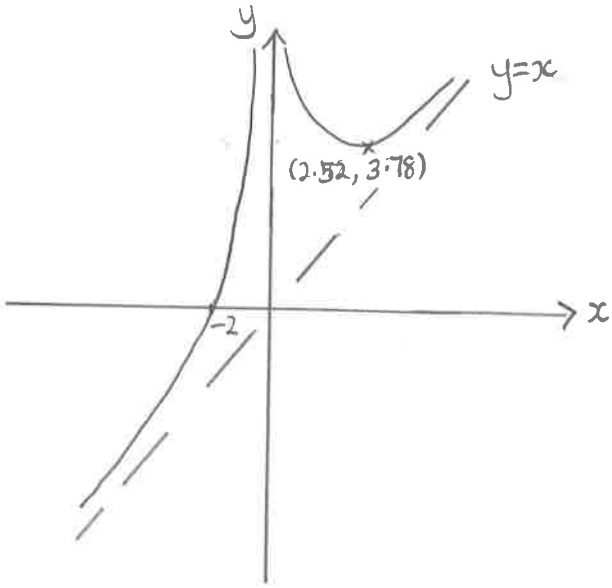
$y' = 2 + \frac{8}{x^2} > 0 \Rightarrow$ no turning pts.

$y = 2x$ is oblique asymptote.

$x \to \infty,\ y \to \infty$

$x \to -\infty,\ y \to -\infty$

(c) $y = x + \frac{8}{x^2}$



$x = 0$ is vertical asymptote

$y = x$ is oblique asymptote

$y = 0,\ x = -2 \qquad (-2, 0)$

$y' = 1 - \frac{16}{x^3} = 0$

$x = \sqrt[3]{16} \approx 2.52$

$y'' = \frac{48}{x^4} \Rightarrow (2.52, 3.78)$

Q2) $y = \dfrac{2x}{x^2+1}$

i) $y' = \dfrac{(x^2+1)(2) - 2x(2x)}{(x^2+1)^2}$

$= \dfrac{-2x^2+2}{(x^2+1)^2}$

$\Rightarrow \ -2x^2+2 > 0$

$x^2 - 1 < 0$

$(x+1)(x-1) < 0$

$-1 < x < 1$ #

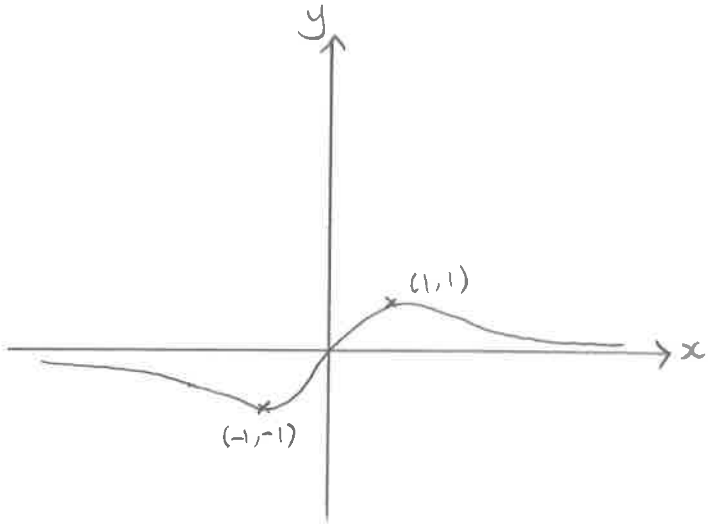
ii) Turning pts: $x = \pm 1$

x	1^-	1	1^+	-1^-	-1	-1^+
$\frac{dy}{dx}$	>0	0	<0	<0	0	>0
sketch	/	—	\	\	—	/

$\Rightarrow$ $(1,1)$ is max pt #

$(-1,-1)$ is min pt. #

(iii)



$y=0$ is horizontal asymptote

$x=0, y=0 \rightarrow$ curve cuts origin

$x \rightarrow \infty, y \rightarrow 0$

$x \rightarrow -\infty, y \rightarrow 0$

Q3) $y = 2(x-1)^3(x+1)$

i) $\dfrac{dy}{dx} = 2(x-1)^3(1) + (x+1)6(x-1)^2$

$= 2(x-1)^2 [x-1 + 3(x+1)]$

$= 2(x-1)^2(4x+2)$

$= 4(x-1)^2(2x+1)$ #

$\dfrac{d^2y}{dx^2} = 4(x-1)^2(2) + (2x+1)8(x-1)$

$= 4(x-1)[2(x-1) + 2(2x+1)]$

$= 4(x-1)(6x)$ #

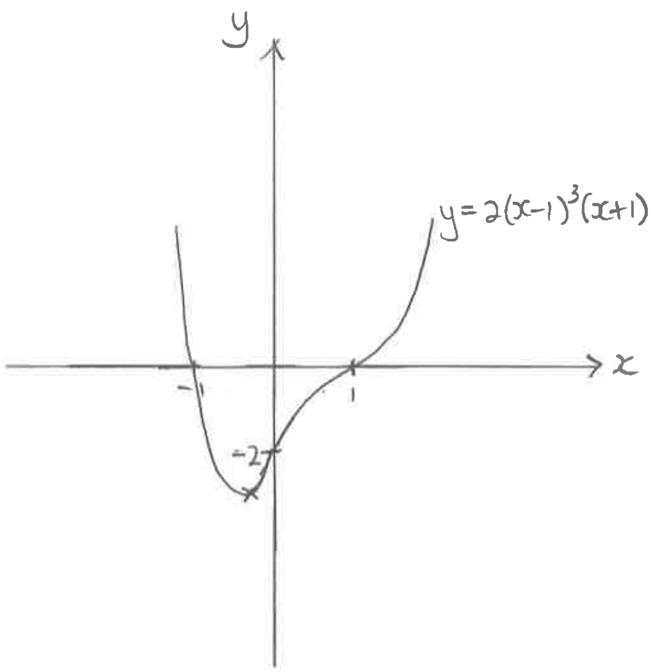
ii) $4(x-1)^2(2x+1) = 0$

$x = 1$ or $-\frac{1}{2}$

$\Rightarrow$ $(-\frac{1}{2}, -\frac{27}{8})$ is min pt #

x	1^-	1	1^+
$\frac{dy}{dx}$	>0	0	>0
sketch	/	—	/

$(1,0)$ is stationary pt of inflexion #



$x=0, y=-2$ $(0,-2)$

$y=0, x=1,-1$ $(1,0)$ and $(-1,0)$

$x \rightarrow \infty, y \rightarrow \infty$

$x \rightarrow -\infty, y \rightarrow \infty$