

Assignment 10 Solutions

Q1(a)

$$\begin{aligned}\lim_{h \rightarrow 0} \frac{(5(x+h)-4)-(5x-4)}{h} &= \lim_{h \rightarrow 0} \frac{5x+5h-4-5x+4}{h} \\ &= \lim_{h \rightarrow 0} 5 \\ &= 5\end{aligned}$$

Q1(b)

$$\begin{aligned}\lim_{h \rightarrow 0} \frac{((x+h)^2+6)-(x^2+6)}{h} &= \lim_{h \rightarrow 0} \frac{x^2+2xh+h^2+6-x^2-6}{h} \\ &= \lim_{h \rightarrow 0} 2x+h \\ &= 2x\end{aligned}$$

Q1(c)

$$\begin{aligned}\lim_{h \rightarrow 0} \frac{\left(2(x+h)-\frac{1}{(x+h)}\right)-\left(2x-\frac{1}{x}\right)}{h} &= \lim_{h \rightarrow 0} \frac{2x+2h-2x-\frac{1}{(x+h)}+\frac{1}{x}}{h} \\ &= \lim_{h \rightarrow 0} \frac{2h-\left(\frac{x-x-h}{x(x+h)}\right)}{h} \\ &= \lim_{h \rightarrow 0} 2+\frac{1}{x(x+h)} \\ &= 2+\frac{1}{x^2}\end{aligned}$$

Q2 (a) $\frac{6x^3-18}{5x^3}$ (b) $-36x^2(2-3x^3)^3$ (c) $-\frac{2}{3}x^{\frac{1}{3}}$ (d) $\frac{2}{x^2}+\frac{10}{x^3}$ (e) $14x^{\frac{5}{2}}-12x^2$

(f) $\frac{2x+3}{2\sqrt{x^2+3x+2}}$ (g) $\frac{-x+8}{4(x-2)^{\frac{5}{2}}}$ (h) $\frac{4}{3}x^{\frac{2}{3}}-\frac{7}{2}x^{\frac{3}{2}}-10x$ (i) $14x^6+30x^4+1-3x^{-2}$

(j) $(x+1)^{-\frac{1}{2}}\left(\frac{3}{2}x-\frac{1}{2x}-\frac{1}{x^2}+1\right)$ (k) $\frac{(9x^2+1)(x^2+1)}{2x^{\frac{1}{2}}}$

(l) $[(2x+1)^2+1]^2[36x^4+24x^3+6x^2+48x+24]$ (m) $\frac{1}{(x+2)^2}$ (n) $\frac{x^2+4x+1}{(x+2)^2}$

Q3

$$y = \sqrt{\frac{1-x}{x^2+3}}$$
$$\frac{dy}{dx} = \frac{1}{2} \left(\frac{1-x}{x^2+3} \right)^{-\frac{1}{2}} \left(\frac{(x^2+3)(-1) - (1-x)(2x)}{(x^2+3)^2} \right)$$
$$= \frac{x^2 - 2x - 3}{2(x^2+3)^{\frac{3}{2}}(1-x)^{\frac{1}{2}}} = 0$$

$$(x-3)(x+1) = 0$$

$$x = -1 \text{ or } 3$$

Q4

$$A = 4r^3 - 3r^2 - 18r + 5$$

$$\frac{dA}{dr} = 12r^2 - 6r - 18 < 0$$

$$2r^2 - r - 3 < 0$$

$$(2r-3)(r+1) < 0$$

$$-1 < r < \frac{3}{2}$$

Q5

$$y = (2x-3)^4(x-4)^5$$

$$\frac{dy}{dx} = (x-4)^5 4(2x-3)^3(2) + (2x-3)^4 5(x-4)^4$$

$$= (2x-3)^3(x-4)^4(8(x-4) + 5(2x-3))$$

$$= (2x-3)^3(x-4)^4(18x-47) = 0$$

$$x = \frac{3}{2}, 4, \frac{47}{18}$$

Q6(i)

$$f(x) = (x+k)(kx+1)^3$$

$$f'(x) = (x+k)3(kx+1)^2k + (kx+1)^3$$

$$= (kx+1)^2(3kx+3k^2+kx+1)$$

$$f'(1) = (k+1)^2(3k+3k^2+k+1) = 0$$

$$(k+1)^2(3k+1)(k+1) = 0$$

$$k = -1 \text{ or } -\frac{1}{3}$$

Q6(ii)

$$k = -1$$

$$f'(2) = (-2+1)^2(-3(2)+3(-1)^2+(-1)2+1)$$

$$= -4$$