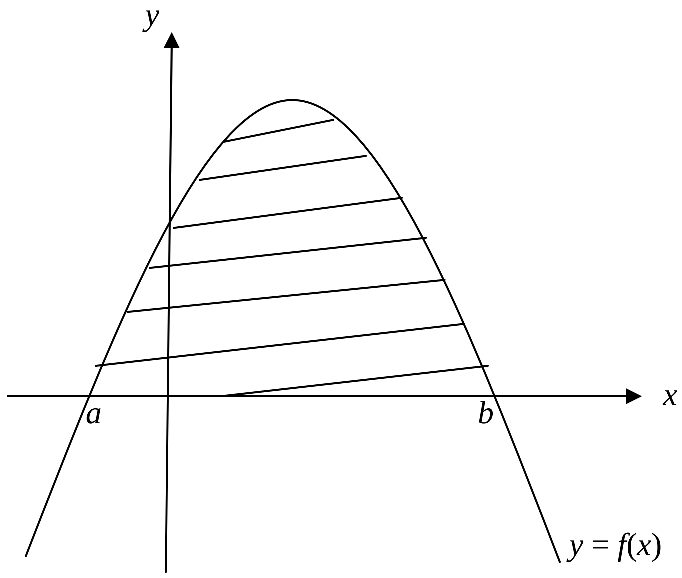
Area of a region using Integration

(1) Area bounded by 1 curve and the *x*-axis (area is <u>above</u> *x*-axis)



Area = $\boxed{\int_a^b f(x)\,dx}$ *where* $y = f(x)$ is the equation of the curve

Example 1: Find the area under the curve $y = (x - 3)^2$ between $x = 3$ and $x = 6$.

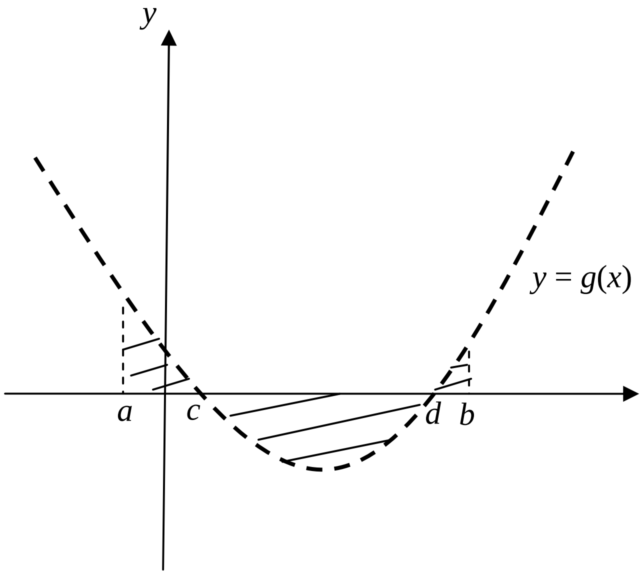
(2) Area bounded by 1 curve and the *x*-axis (area is <u>below</u> *x*-axis)

Example 2: Find the area of the region bounded by the curve $y = x^2 - 4x + 3$ and the *x*-axis.

**Take the absolute value (modulus) of the Area if the answer is negative.

In other words, if the area is below the *x*-axis, Area = $\boxed{-\int_a^b f(x)\,dx}$ *where* $y = f(x)$ is the equation of the curve

(3) Area bounded by 1 curve and the *x*-axis (some parts above, some below *x*-axis)



Area = $\boxed{\int_a^c g(x)\,dx - \int_c^d g(x)\,dx + \int_d^b g(x)\,dx} \neq \int_a^b \boldsymbol{g(x)\,dx}$

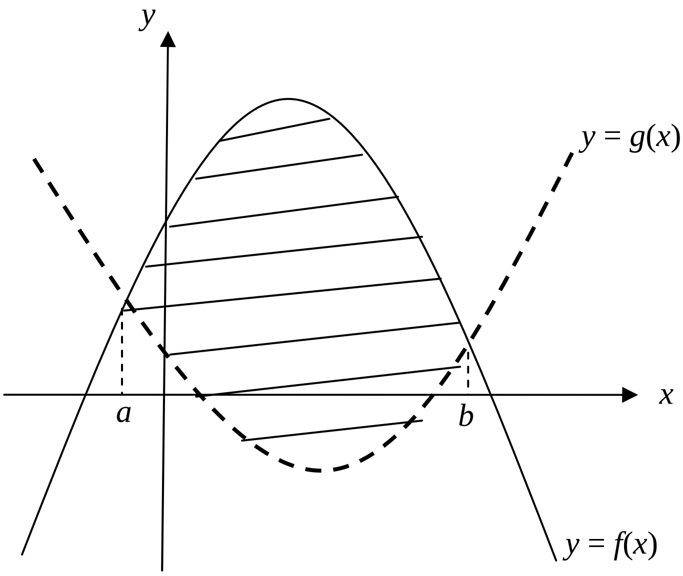
** If the areas you are required to find – one part is above the *x*-axis and the other is below, you will need to split them up into 2 pieces, find their areas separately, then add the areas together.

Example 3: Find the area of the region bounded by the curve $y = x^3 - x^2 - 6x$ and the *x*- axis.

(4) Area bounded by 2 curves

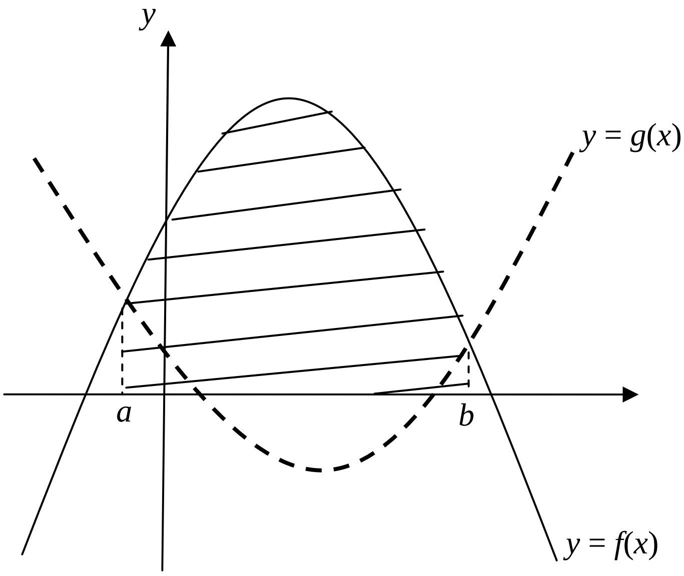
To find the area bounded by $y = f(x)$ and $y = g(x)$, first find the points of intersection of the 2 curves, a and b.

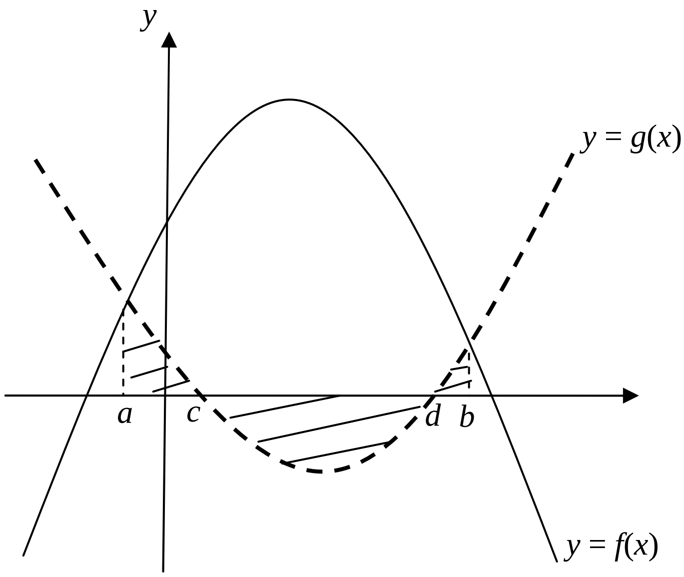
Area = $\boxed{\int_a^b \left[f(x) - g(x) \right] dx}$, where $y = f(x)$ is the upper curve and $y = g(x)$ is the lower curve.



Shaded area → $\int_a^b f(x)\, dx$

Shaded area→ $\int_a^c g(x)\, dx - \int_c^d g(x)\, dx + \int_d^b g(x)\, dx$





Therefore, area bounded by the 2 curves = $\int_a^b f(x)\, dx - \int_a^c g(x)\, dx$

$$+ \left[-\int_c^d g(x)\, dx \right]$$

$$-\int_d^b g(x)\, dx$$

$$\int_a^b f(x)\, dx - \left[\int_a^c g(x)\, dx + \int_c^d g(x)\, dx + \int_d^b g(x)\, dx \right]$$

$$= \int_a^b f(x)\, dx - \int_a^b g(x)\, dx$$

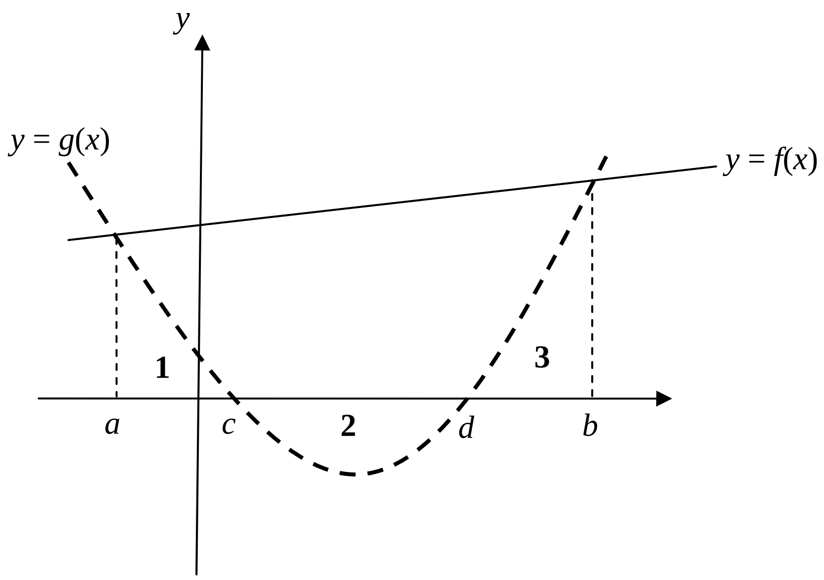
$$= \int_a^b f(x) - g(x)\, dx$$

(4A) Area bounded by curve and line

Method is similar to case (4) area bounded by 2 curves

To find the area bounded by $y = f(x)$ and $y = g(x)$, first find the points of intersection of the curve and line, a and b.

Area = $\boxed{\int_a^b \left[f(x) - g(x) \right] dx}$, where $y = f(x)$ is the upper curve and $y = g(x)$ is the lower curve.



Area bounded by $y = f(x)$ will be the trapezium, or simply $\int_a^b f(x)\, dx$.

Area bounded by $y = g(x)$ will be the regions (1), (2) and (3), which is $\int_a^c g(x)\, dx - \int_c^d g(x)\, dx + \int_d^b g(x)\, dx$

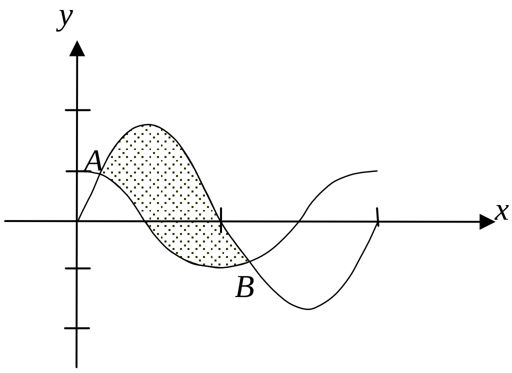
Hence, area bounded by the line and the curve is

Area of Trapezium – Area of region (1) – Area of region (3) + Area of region (2)

$$\int_a^b f(x)\, dx \quad - \quad \int_a^c g(x)\, dx \quad - \quad \int_d^b g(x)\, dx \quad + \left[-\int_c^d g(x)\, dx \right]$$

Which is $\int_a^b f(x)\, dx - \left[\int_a^c g(x)\, dx + \int_c^d g(x)\, dx + \int_d^b g(x)\, dx \right] = \boxed{\int_a^b \left[f(x) - g(x) \right] dx}$

Example 4: The diagram shows the curves $y = \sqrt{3}\sin x$ and $y = \cos x$ for $0 \le x \le 2\pi$, intersecting at the points A and B.



(i) Find the coordinates of A and B.

(ii)Find the area of the shaded region bounded by the two curves.

Example 5: Find the area bounded by the curve $y = 2\sin 2x$, the line $y = \frac{2}{\pi}x - 1$ and the y-axis.