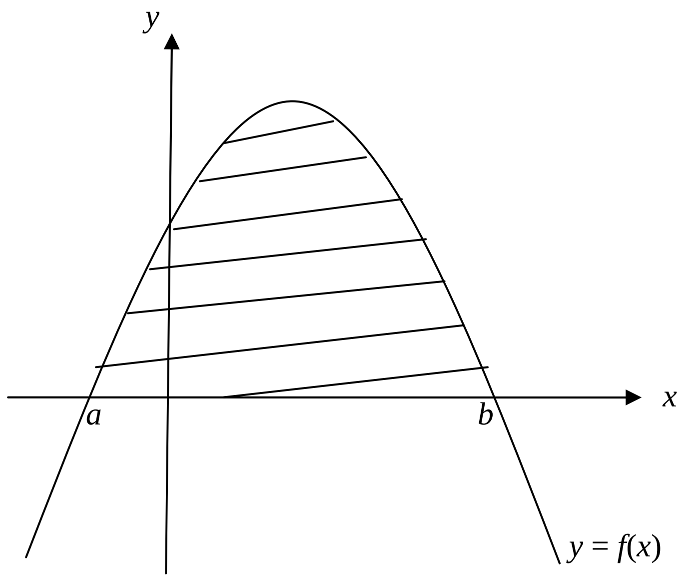
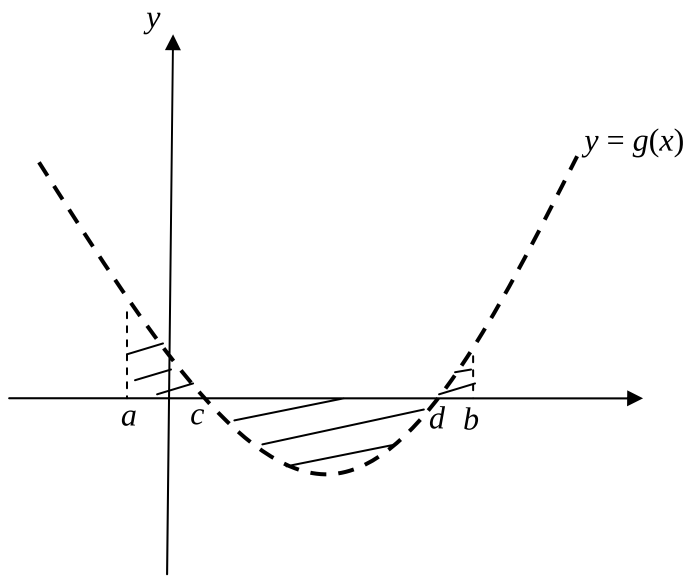
Area bounded by 1 curve and the *x*-axis



Area = $\boxed{\int_a^b f(x)\,dx}$

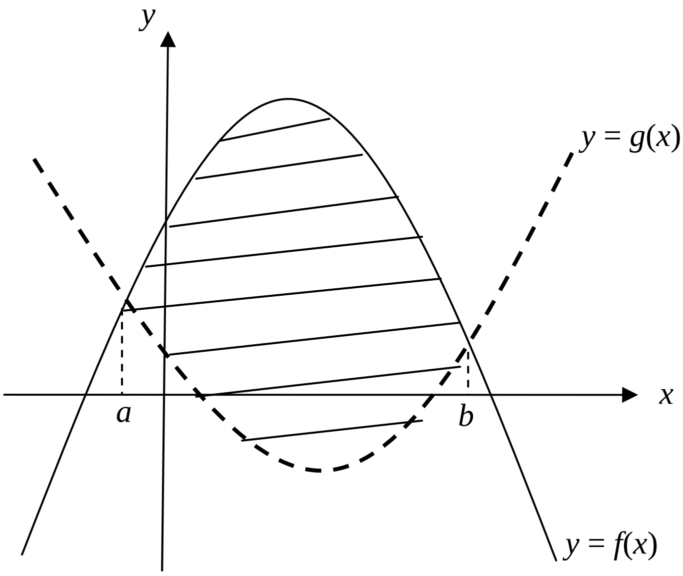


Area = $\boxed{\int_a^c g(x)\,dx - \int_c^d g(x)\,dx + \int_d^b g(x)\,dx} \neq \int_a^b g(x)\,dx$

Area bounded by 2 curves

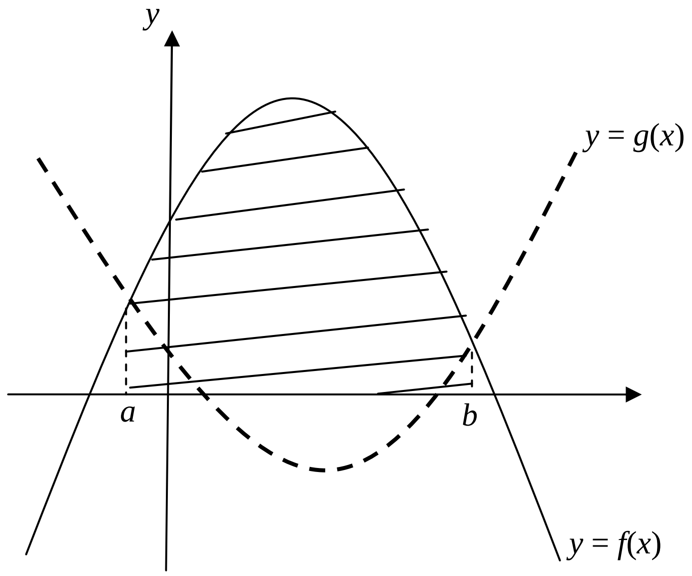
To find the area bounded by $y = f(x)$ and $y = g(x)$, first find the points of intersection of the 2 curves, a and b.

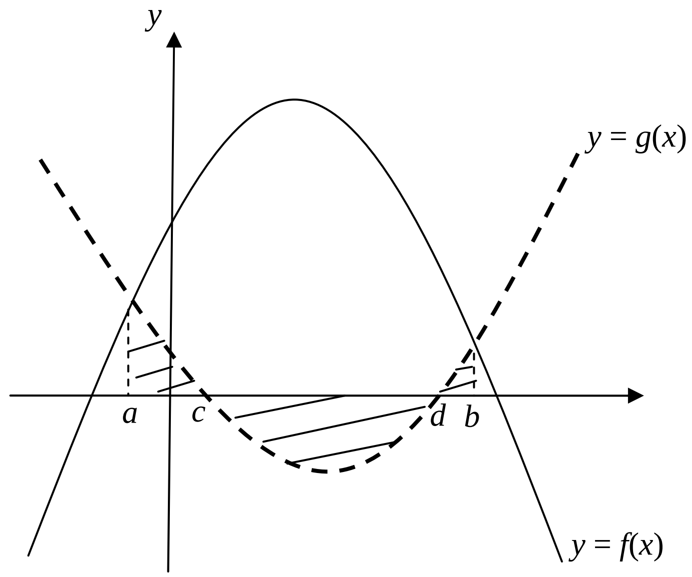
Area = $\boxed{\int_a^b \left[f(x) - g(x) \right] dx}$, where $y = f(x)$ is the upper boundary and $y = g(x)$ is the lower boundary.



Shaded area → $\int_a^b f(x)\, dx$

Shaded area → $\int_a^c g(x)\, dx - \int_c^d g(x)\, dx + \int_d^b g(x)\, dx$





Therefore, area bounded by the 2 curves = $\int_a^b f(x)\, dx - \int_a^c g(x)\, dx$

$+\left[-\int_c^d g(x)\, dx \right]$

$-\int_d^b g(x)\, dx$

$$\int_a^b f(x)\, dx - \left[\int_a^c g(x)\, dx + \int_c^d g(x)\, dx + \int_d^b g(x)\, dx \right]$$

$$= \int_a^b f(x)\, dx - \int_a^b g(x)\, dx$$

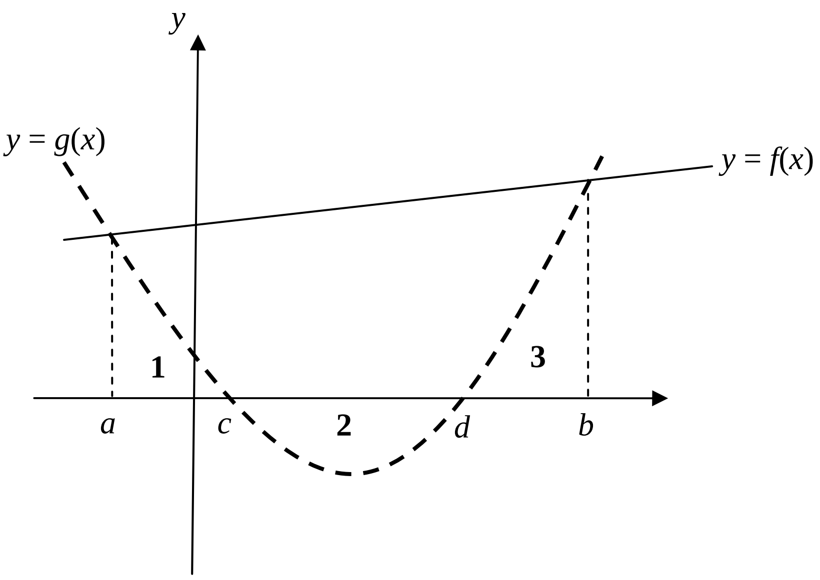
$$= \int_a^b f(x) - g(x)\, dx$$

Area bounded by curve and line

Method is similar to area bounded by 2 curves

To find the area bounded by $y = f(x)$ and $y = g(x)$, first find the points of intersection of the curve and line, a and b.

Area = $\boxed{\int_a^b \left[f(x) - g(x) \right] dx}$, where $y = f(x)$ is the upper boundary and $y = g(x)$ is the lower boundary.



Area bounded by $y = f(x)$ will be the trapezium, or simply $\int_a^b f(x)\,dx$.

Area bounded by $y = g(x)$ will be the regions (1), (2) and (3), which is $\int_a^c g(x)\,dx - \int_c^d g(x)\,dx + \int_d^b g(x)\,dx$

Hence, area bounded by the line and the curve is

Area of Trapezium – Area of region (1) – Area of region (3) + Area of region (2)

$$\int_a^b f(x)\,dx \quad - \quad \int_a^c g(x)\,dx \quad - \quad \int_d^b g(x)\,dx \quad + \quad \left[-\int_c^d g(x)\,dx \right]$$

Which is $\int_a^b f(x)\,dx - \left[\int_a^c g(x)\,dx + \int_c^d g(x)\,dx + \int_d^b g(x)\,dx \right] = \boxed{\int_a^b \left[f(x) - g(x) \right] dx}$