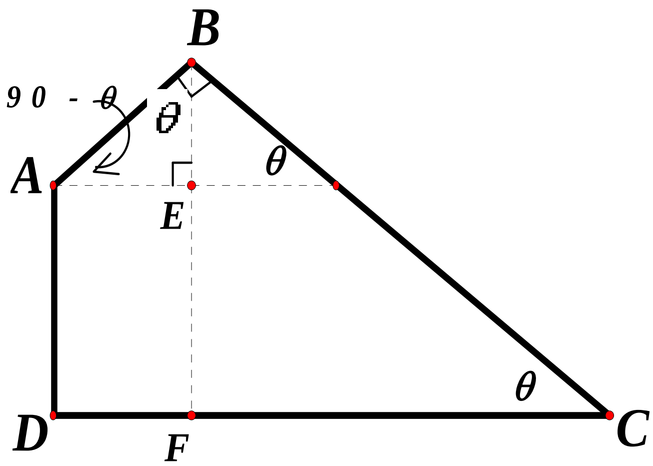
Test 2 Solutions (SMTP&SBGE)_Ver 2

1(i)	$\cos(A-B)=\frac{2}{5}$ $\cos A\cos B+\sin A\sin B=\frac{2}{5}$ $\sin A\sin B=\frac{2}{5}-\frac{2}{3}=\frac{-4}{15}$
(ii)	$\cot A\cot B=\frac{\cos A}{\sin A}\times\frac{\cos B}{\sin B}$ $=\frac{\frac{2}{3}}{-\frac{4}{15}}$ $=-\frac{5}{2}$
2(a)(i)	$\cos 75^\circ=\cos(30^\circ+45^\circ)$ $=\cos 30^\circ\cos 45^\circ-\sin 30^\circ\sin 45^\circ$ $=\frac{\sqrt{3}}{2}\left(\frac{\sqrt{2}}{2}\right)-\frac{1}{2}\left(\frac{\sqrt{2}}{2}\right)$ $=\frac{\sqrt{6}-\sqrt{2}}{4}$ (shown)
(ii)	$\sin(-15^\circ)=-\sin 15^\circ$ $=-\cos 75^\circ$ $=\frac{\sqrt{2}-\sqrt{6}}{4}$
3	$\sin 2\theta+\sin 4\theta+\cos 2\theta+\cos 4\theta$ $=2\sin 3\theta\cos\theta+2\cos 3\theta\cos\theta$

	$= 2\cos\theta\left(\sqrt{2}\sin\left(3\theta + \tan^{-1}\left(\frac{1}{1}\right)\right)\right)$ $= 2\cos\theta(\sin 3\theta + \cos 3\theta) = 2\cos\theta\left(\sqrt{2}\sin\left(3\theta + \frac{\pi}{4}\right)\right)$ $= 2\sqrt{2}\cos\theta\sin\left(3\theta + \frac{\pi}{4}\right)$ (*proven*)
	$\sin 2\theta + \sin 4\theta + \cos 2\theta + \cos 4\theta = 0$ $2\sqrt{2}\cos\theta\sin\left(3\theta + \frac{\pi}{4}\right) = 0$ $\cos\theta\sin\left(3\theta + \frac{\pi}{4}\right) = 0$ $0 \le \theta \le \pi$ $\frac{\pi}{4} \le 3\theta + \frac{\pi}{4} \le \frac{13\pi}{4}$ $\cos\theta = 0 \Rightarrow \theta = \frac{\pi}{2}$ $(or\ 1.57)$ $or\ \sin\left(3\theta + \frac{\pi}{4}\right) = 0$ $3\theta + \frac{\pi}{4} = \pi,\ 2\pi,\ 3\pi$ $\Rightarrow \theta = \frac{\pi}{4},\ \frac{7\pi}{12},\ \frac{11\pi}{12}$ $(or\ 0.785, 1.83, 2.88)$ $\therefore\ \theta = \frac{\pi}{4},\ \frac{\pi}{2},\ \frac{7\pi}{12},\ \frac{11\pi}{12}$ $(or\ 0.785, 1.57, 1.83, 2.88)$
4(a)	$16\sin^2 x\cos^2 x = 1$ $\Rightarrow 4\sin x\cos x = \pm 1$ $\Rightarrow 2(2\sin x\cos x) = \pm 1$ $\therefore \sin 2x = \pm\frac{1}{2}$ $\alpha = 30°$ *2x lies in all quadrants* $2x = 30°, 150°, 210°, 330°, 390°, 510°, 570°, 690°$ $x = 15°, 75°, 105°, 165°, 195°, 255°, 285°, 345°$
(b)	$\tan z\tan 2z = 8$,

	$Let\ t=\tan z$ $t\left(\frac{2t}{1-t^2}\right)=8$ $\frac{2t^2}{1-t^2}=8$ $2t^2=8-8t^2$ $10t^2=8$ $t^2=\frac{4}{5} \Rightarrow \tan^2 z=\frac{4}{5}$ $t=\pm\sqrt{\frac{4}{5}}$ $\therefore \tan z=\pm\sqrt{\frac{4}{5}}$ $\alpha=41.81^\circ$ $\therefore z\approx 41.8^\circ, 138.2^\circ, 221.8^\circ, 318.2^\circ$
(c)	$\sin y-\sin 10y\sin 8y=\cos 10y\cos 8y$ $\sin y=\cos 10y\cos 8y+\sin 10y\sin 8y$ $\sin y=\cos(10y-8y)$ $\sin y=\cos 2y$ $\cos 2y-\sin y=0$ $1-2\sin^2 y-\sin y=0$ $2\sin^2 y+\sin y-1=0$ $(2\sin y-1)(\sin y+1)=0$ $\sin y=\frac{1}{2}\ or\ \sin y=-1$ $y=30°,150°,270°$
5(i)	$\tan A=\frac{3}{h},\ \tan B=\frac{5}{h}$
5(ii)	$\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$

	$\tan 45^\circ = \dfrac{\frac{3}{h}+\frac{5}{h}}{1-\frac{3}{h}\times\frac{5}{h}}$ $1 = \dfrac{\frac{8}{h}}{\frac{h^2-15}{h^2}}$ $\dfrac{h^2-15}{h^2} = \dfrac{8}{h}$ $h^2-15 = 8h$ $h^2-8h-15 = 0$ $h = \dfrac{-(-8)\pm\sqrt{(-8)^2-4(1)(-15)}}{2(1)}$ $= \dfrac{8\pm\sqrt{124}}{2}$ $= \dfrac{8\pm 2\sqrt{31}}{2}$ $= 4+\sqrt{31} \ \left(reject\ \ 4-\sqrt{31} \ \because h>0\right)$
6(i)	$\angle ABE = \theta$ $\sin\theta = \dfrac{AE}{AB} = \dfrac{AE}{1} \Rightarrow AE = \sin\theta$ $\cos\theta = \dfrac{BE}{AB} = \dfrac{BE}{1} \Rightarrow BE = \cos\theta$ *Perimeter* $= AB+BC+CD+DA$ $=1+7+(CF+FD)+(BF-BE)$ $=8+(CF+AE)+(BF-BE)$ $=8+(7\cos\theta+\sin\theta)+(7\sin\theta-\cos\theta)$ $=8+6\cos\theta+8\sin\theta$

(ii)	 $Perimeter = 8 + 10\cos\left[\theta - \tan^{-1}\left(\frac{4}{3}\right)\right]$ $= 8 + 10\cos(\theta - 53.13°)$ The maximum value of the perimeter is 18 cm
(iii)	Largest value of $1 - 2P + P^2 = (1 - P)^2$ $= [1 - (8 + 10\cos(\theta - \tan^{-1}\frac{4}{3})]$ $= \{1 - [8 + 10(1)]\}^2 \quad = (-17)^2$ $= 289$

Bonus Questions

BQ1

$$Let\quad \tan^{-1}\frac{1}{2}=A,\ \ \tan^{-1}\frac{1}{5}=B,\ \ \tan^{-1}\frac{1}{8}=C$$

$$\Rightarrow \tan A=\frac{1}{2},\tan B=\frac{1}{5},\tan C=\frac{1}{8}$$

$$\tan(A+B+C)=\frac{\tan\left(A+B\right)+\tan C}{1-\tan(A+B)\tan C}=\frac{\dfrac{\left(\frac{1}{2}\right)+\left(\frac{1}{5}\right)}{1-\left(\frac{1}{2}\right)\left(\frac{1}{5}\right)}+\dfrac{1}{8}}{1-\left[\dfrac{\left(\frac{1}{2}\right)+\left(\frac{1}{5}\right)}{1-\left(\frac{1}{2}\right)\left(\frac{1}{5}\right)}\right]\left(\dfrac{1}{8}\right)}=1$$

$$\begin{aligned}\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{8}&=A+B+C\\&=\tan^{-1}1\\&=45^{0}\end{aligned}$$

BQ2

$$\cot(A+B)$$

$$=\frac{1-\tan A\tan B}{\tan A+\tan B}$$

$$=\frac{\cot A\cot B-1}{\cot B+\cot A}$$

$$=1$$

$$\cot A+\cot B=\cot A\cot B-1$$

$$\therefore \cot A+\cot B-\cot A\cot B=-1$$

From $\cot A+\cot B-\cot A\cot B=-1$, we have

$$\cot A\cot B-\cot A-\cot B=1$$

$$\left(\cot 25^\circ-1\right)\left(\cot 24^\circ-1\right)\left(\cot 23^\circ-1\right)\left(\cot 22^\circ-1\right)\left(\cot 21^\circ-1\right)\left(\cot 20^\circ-1\right)$$

$$=\left[\left(\cot 25^\circ-1\right)\left(\cot 20^\circ-1\right)\right]\left[\left(\cot 24^\circ-1\right)\left(\cot 21^\circ-1\right)\right]\left[\left(\cot 23^\circ-1\right)\left(\cot 22^\circ-1\right)\right]$$

$$=\left(1-\cot 25^\circ-\cot 20^\circ+\cot 25^\circ\cot 20^\circ\right)\left(1-\cot 24^\circ-\cot 21^\circ+\cot 24^\circ\cot 21^\circ\right)\left(1-\cot 23^\circ-\cot 22^\circ+\cot 23^\circ\cot 22^\circ\right)$$

$$=(1+1)(1+1)(1+1)$$

$$=8$$