Test 2 Solutions (SMTP&SBGE)_Ver 1

1(i)	A and B lies in the 2nd quad $\Rightarrow \cos A = -\frac{12}{13}, \tan A = -\frac{5}{12}$ and $\sin B = \frac{4}{5}, \cos B = -\frac{3}{5}$ $\sin(A-B) = \sin A \cos B - \sin B \cos A$ $= (\frac{5}{13})(-\frac{3}{5}) - (\frac{4}{5})(-\frac{12}{13})$ $= \frac{33}{65}$
(ii)	$\cos B = 2\cos^2 \frac{1}{2}B - 1$ $\cos^2 \frac{1}{2}B = \frac{\cos B + 1}{2}$ $= \frac{-\frac{3}{5}+1}{2} = \frac{1}{5}$ $\cos \frac{1}{2}B = \pm\sqrt{\frac{1}{5}}$ $= \frac{\sqrt{5}}{5} \quad (\because \cos \frac{1}{2}B > 0)$
2	$\cos 2x + \cos 4x + \cos 6x + 1 =$ $\cos 2x + \cos 4x + 2\cos^2 3x$ $= 2\cos\left(\frac{2x+4x}{2}\right)\cos\left(\frac{4x-2x}{2}\right) + 2\cos^2 3x$ $= 2\cos 3x \cos x + 2\cos^2 3x$ $= 2\cos 3x(\cos x + \cos 3x)$ $= 2\cos 3x(2\cos\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right)$ $= 4\cos 3x \cos 2x \cos x$

	$\cos 15° = \frac{\cos 2(15°) + \cos 4(15°) + \cos 6(15°) + 1}{4\cos 3(15°)\cos 2(15°)}$ $= \frac{\frac{\sqrt{3}}{2} + \frac{1}{2} + 0 + 1}{4\left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right)}$ $= \frac{\sqrt{6} + \sqrt{2}}{4}$
3	$\frac{\sin(A-B)}{\sin(A+B)} = \frac{3}{2}$ $\frac{\sin A\cos B - \cos A\sin B}{\sin A\cos B + \cos A\sin B} = \frac{3}{2}$ $2\sin A\cos B - 2\cos A\sin B = 3\sin A\cos B + 3\cos A\sin B$ $-\sin A\cos B = 5\cos A\sin B$ $-\tan A = 5\tan B$ $\tan A + 5\tan B = 0$
4(a)	$3\sin 2\theta = \sin\theta$ $6\sin\theta\cos\theta - \sin\theta = 0$ $\sin\theta(6\cos\theta - 1) = 0$ $\sin\theta = 0 \quad or \quad \cos\theta = \frac{1}{6}$ $\theta = 180° \quad or \quad \theta = 80.4°, 279.6°$
4(b)	$3\cos(x + 40°) = 2\sin x$ $3(\cos x\cos 40° - \sin x\sin 40°) = 2\sin x$ $\cos x(3\cos 40°) = \sin x(2 + 3\sin 40°)$ $\tan x = \frac{3\cos 40°}{2 + 3\sin 40°}$ $\alpha = 30.33°$ $x = 30.3°, 210.3°$

4(c)	$8\cos^2\theta - 6\sin 2\theta = 1$ $8\left(\frac{\cos 2\theta + 1}{2}\right) - 6\sin 2\theta = 1$ $4\cos 2\theta + 4 - 6\sin 2\theta - 1 = 0$ $4\cos 2\theta - 6\sin 2\theta + 3 = 0$ $\sqrt{4^2+6^2}\cos\left(2\theta + 56.31°\right) = -3$ $\cos\left(2\theta + 56.31°\right) = -\frac{3}{\sqrt{52}}$ $basic\ angle = 65.42°$ $2\theta + 56.31° = 114.58°, 245.42°, 474.58, 605.42°$ $\theta = 29.1°, 94.6°, 209.1°, 274.6°$
5	$\tan(x+45°)$ $= \tan\left[(x+y) - (y-45°)\right]$ $= \frac{\tan(x+y) - \tan(y-45°)}{1+\tan(x+y)\tan(y-45°)}$ $= \frac{-\frac{3}{5} - \frac{1}{3}}{1+\left(-\frac{3}{5}\right)\left(\frac{1}{3}\right)}$ $= -\frac{7}{6}$
6	$BC = 5\cos x°$ $EC = AC - AE$ $= 5\sin x° - 2\sin x°$ $= 3\sin x°$ $DE = 2\cos x°$ $\therefore P = BC + EC + 2DE + AE + AB$ $= 5\cos x° + 3\sin x° + 4\cos x° + 2\sin x° + AB$ $= 9\cos x° + 5\sin x° + 1$ (*shown*)
(i)	$9\cos x° + 5\sin x° = \sqrt{9^2+5^2}\cos(x-\alpha)$ $\therefore R = \sqrt{106}$ $\tan\alpha = \frac{5}{9} \Rightarrow \alpha = 29.05° \approx 29.1°$ $\therefore 9\cos x° + 5\sin x° = \sqrt{106}\cos(x - 29.05°)$
(ii)	$P_{max} = \sqrt{106} + 1$ (or 11.3) $\cos(x - 29.05) = 1$ $x - 29.05° = 0°,\ 360°(n.a.)$ $x = 29.1°$

(iii)	$\frac{1-P^2}{1+P^2}=-1+\frac{2}{1+P^2}$ Maximum value of $\frac{1-P^2}{1+P^2}=-1+\frac{2}{1+(0)^2}=1$
BQ1	**Bonus Questions** $\tan^{-1}1=a \Rightarrow 1=\tan a,\ \tan^{-1}2=b \Rightarrow \tan b=2$ $\tan^{-1}3=c \Rightarrow 3=\tan c,\ \tan^{-1}7=d \Rightarrow \tan d=7$ $\tan(a+b)=\frac{\tan a+\tan b}{1-\tan a\tan b}$ $=\frac{1+2}{1-1(2)}=-3$ $\tan(c+d)=\frac{3+7}{1-3(7)}$ $=-\frac{1}{2}$ $\tan(\tan^{-1}1+\tan^{-1}2+\tan^{-1}3+\tan^{-1}7)$ $=\tan(a+b+c+d)$ $=\frac{\tan(a+b)+\tan(c+d)}{1-\tan(a+b)\tan(c+d)}$ $=\frac{-3-\frac{1}{2}}{1-(-3)(-\frac{1}{2})}$ $=7$
BQ2	$\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}=1$ $\tan A+\tan B=1-\tan A\tan B$ $\therefore \tan A+\tan B+\tan A\tan B=1$ $\left(1+\tan 16^\circ\right)\left(1+\tan 19^\circ\right)\left(1+\tan 26^\circ\right)\left(1+\tan 29^\circ\right)$ $=\left(1+\tan 16^\circ\right)\left(1+\tan 29^\circ\right)\left(1+\tan 19^\circ\right)\left(1+\tan 26^\circ\right)$ $=\left(1+\tan 16^\circ+\tan 29^\circ+\tan 16^\circ\tan 29^\circ\right)\left(1+\tan 19^\circ+\tan 26^\circ+\tan 19^\circ\tan 26^\circ\right)$ $=(1+1)(1+1)$ $=4$