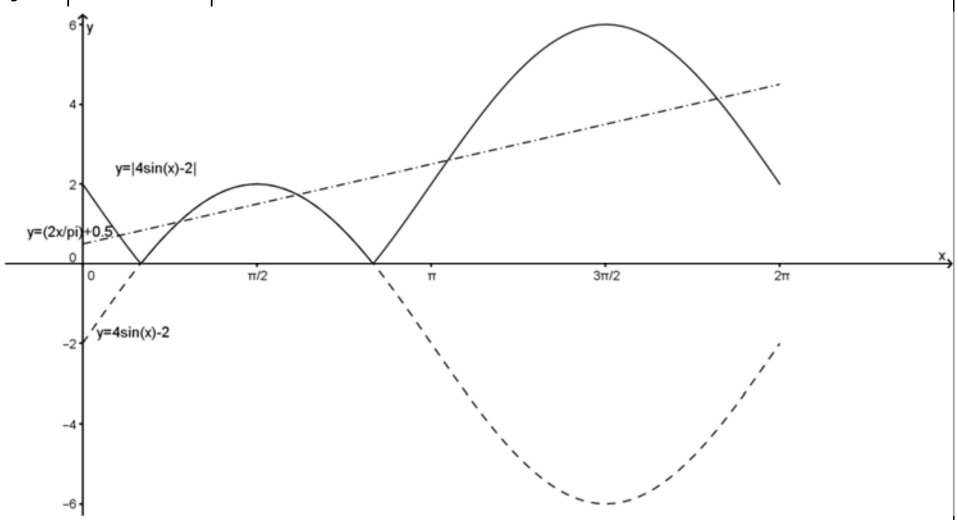
Solutions (SMTP&SBGE)_ver 1 (27012015)

1(a)	$\cos(2x-60°)=\sin 30°$ $\cos(2x-60°)=\frac{1}{2}$ $\alpha=60°$ $-60°\le 2x-60°\le 660°$ $2x-60°=60°,300°,420°,660°,-60°$ $x=0°,60°,180°,240°,360°$
1 (b)	$2\cot\theta-\sqrt{3}\tan\theta=2-\sqrt{3}$ $\frac{2}{\tan\theta}-\sqrt{3}\tan\theta=2-\sqrt{3}$ $\sqrt{3}\tan^2\theta+\left(2-\sqrt{3}\right)\tan\theta-2=0$ $\left(\sqrt{3}\tan\theta+2\right)\left(\tan\theta-1\right)=0$ $\tan\theta=1 \text{ or } \tan\theta=-\frac{2}{\sqrt{3}}$ $\alpha=\frac{\pi}{4} \text{ or } \alpha=0.85707...$ $\theta=\frac{\pi}{4},\frac{5\pi}{4},2.28452...,5.42611...$ $\theta=\frac{\pi}{4},\frac{5\pi}{4},1.99,5.13$ $=0.785,2.28,3.93,5.43$
2(i)	$\frac{1-\tan^2 2x}{\sec^2 2x+2\tan 2x}=\frac{1-\frac{\sin^2 2x}{\cos^2 2x}}{\frac{1}{\cos^2 2x}+\frac{2\sin 2x}{\cos 2x}}$ $=\frac{\cos^2 2x-\sin^2 2x}{1+2\sin 2x\cos 2x}$ $=\frac{\left(\cos 2x+\sin 2x\right)\left(\cos 2x-\sin 2x\right)}{\sin^2 2x+\cos^2 2x+2\sin 2x\cos 2x}$ $=\frac{\left(\cos 2x+\sin 2x\right)\left(\cos 2x-\sin 2x\right)}{\left(\sin 2x+\cos 2x\right)^2}$ $=\frac{\cos 2x-\sin 2x}{\cos 2x+\sin 2x}$

2(ii)	$\frac{\cos 2x-\sin 2x}{\cos 2x+\sin 2x}=-\frac{3}{2}$ $-2\cos 2x+2\sin 2x=3\cos 2x+3\sin 2x$ $\sin 2x=-5\cos 2x$ $\tan 2x=-5$ $\alpha=1.3734...$ $2x=\pi-\alpha, 2\pi-\alpha, 3\pi-\alpha, 4\pi-\alpha$ $x=0.88409..., 2.45489..., 4.02568...(rej), 5.59648...(rej)$ $\approx 0.884, 2.45$
3	$f(x)=4\cos x-2 \rightarrow a=4, b=-2$ $y=\lvert 4\sin x-2\rvert \rightarrow$ curve to sketch  5 roots

4	$\dfrac{\sin^2\beta}{5\cos^2\beta-1}=\dfrac{4}{5}$ $5\sin^2\beta=20\cos^2\beta-4$ $5\sin^2\beta=20\left(1-\sin^2\beta\right)-4$ $5\sin^2\beta=16-20\sin^2\beta$ $25\sin^2\beta=16$ $\sin^2\beta=\dfrac{16}{25}$ $\sin\beta=\dfrac{4}{5}$ since $90^\circ\le\beta<180^\circ$ $\cos\beta=-\dfrac{3}{5}$ $\dfrac{\sin\beta}{5\cos\beta-1}=\dfrac{\frac{4}{5}}{5\left(-\frac{3}{5}\right)-1}$ $=-\dfrac{1}{5}$
5(i)	$\tan\dfrac{\pi}{6}=\dfrac{2}{PX}\rightarrow PX=2\sqrt{3}$ cm Perimeter = 2*PX* + Arc *XY* $=4\sqrt{3}+\dfrac{4\pi}{3}$cm
5(ii)	Area of *PQR* $=\dfrac{1}{2}\left(4\sqrt{3}+4\right)\left(4\sqrt{3}+4\right)\sin\dfrac{\pi}{3}$ $=24+16\sqrt{3}$ cm^2
5(iii)	Area of unshaded regions $=24+16\sqrt{3}-\left[\dfrac{1}{2}(4)(4)\sin\dfrac{\pi}{3}\right]-3\left[\dfrac{1}{2}(2)^2\dfrac{5\pi}{3}\right]$ $=13.36868...$ ≈ 13.4 cm^2
6	$BM=r\sin\dfrac{\pi}{3}\therefore BC=2r\sin\dfrac{\pi}{3}$ Area of shaded region = Area of segment *AQB* – Area of segment *APB*

	$=\frac{1}{2}r^2\left(\frac{2\pi}{3}-\sin\frac{2\pi}{3}\right)-\frac{1}{2}\left(2r\sin\frac{\pi}{3}\right)^2\left(\frac{\pi}{3}-\sin\frac{\pi}{3}\right)$ $=\frac{1}{2}r^2\left(\frac{2\pi}{3}-\frac{\sqrt{3}}{2}\right)-\frac{1}{2}\left(4r^2\left(\frac{3}{4}\right)\right)\left(\frac{\pi}{3}-\frac{\sqrt{3}}{2}\right)$ $=\frac{1}{2}r^2\left(\frac{2\pi}{3}-\frac{\sqrt{3}}{2}\right)-\frac{3}{2}r^2\left(\frac{\pi}{3}-\frac{\sqrt{3}}{2}\right)$ $=r^2\left(\frac{\pi}{3}-\frac{\sqrt{3}}{4}-\frac{\pi}{2}+\frac{3\sqrt{3}}{4}\right)$ $=r^2\left(\frac{\sqrt{3}}{2}-\frac{\pi}{6}\right)$
BQ	$\sin x+\cos x=\frac{\sqrt{2}}{3}$ $1+2\sin x\cos x=\frac{2}{9}$ $2\sin x\cos x=-\frac{7}{9}$ $(\sin x-\cos x)^2=1-2\sin x\cos x$ $=1-\left(-\frac{7}{9}\right)$ $=\frac{16}{9}$ Since $270^\circ\le x\le 360^\circ$, $\sin x-\cos x=-\frac{4}{3}$ $\frac{\sin x+\cos x}{\sin x-\cos x}=\frac{\sqrt{2}}{3}\div-\frac{4}{3}$ $\frac{\tan x+1}{\tan x-1}=-\frac{\sqrt{2}}{4}$ $4\tan x+4=-\sqrt{2}\tan x+\sqrt{2}$ $\tan x\left(4+\sqrt{2}\right)=\sqrt{2}-4$ $\tan x=\frac{\sqrt{2}-4}{\sqrt{2}+4}$