Name: ______________________________ (   )   Class: _________   Date: *28 Jan 2015*

Circular Measure and Trigonometry **Class Test 1 –V2**
1 hour **40 marks**

Instructions:

- Answer all the questions.
- Omission of essential working will result in loss of marks.
- You are reminded of the need for clear presentation in your answers.
- If the numerical answer is not exact, give your answer to 1 decimal place for angles in degrees and 3 significant figures for all others unless otherwise stated in the question.
- The number of marks is given in brackets [   ] at the end of each question or part question.

1. Without using a calculator, evaluate $\dfrac{2\sin 60° - \tan 30°}{\cos 135° \operatorname{cosec} 45° + \cot 60°}$. Show your working clearly.

[4]

2. Find all angles which satisfy the equations

(a) $\sin(2x + 40°) = 3\cos(50° - 2x) - 1$, for $0 \le x \le 360°$, [4]

(b) $\sqrt{3}\sin^2\theta - 2\sin\theta\cos\theta - \sqrt{3}\cos^2\theta = 0$, for $0 \le \theta \le 2\pi$. [4]

3. Prove that $(1 + \sin x - \cos x)^2 \equiv 2(1 + \sin x)(1 - \cos x)$. [3]

Hence

(i) prove that $(1 - \sin x - \cos x)^2 \equiv 2(1 - \sin x)(1 - \cos x)$. [1]

(ii) find all values of x between 0 and 2π inclusive for which $(1 + \sin x - \cos x)^2 = 1 - \cos x$.

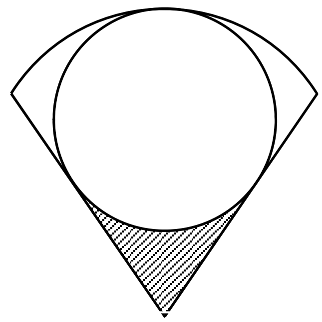
[4]

4. (i) It is given that the function $\mathrm{f}(x) = a\sin x + 2b$, where $0 \le x \le 2\pi$ and $a > 0$, $b > 0$ has a maximum value of 5 and a minimum value of −1, find the value of a and of b.

(ii) Using the values of a and b found in (i), sketch the graph of $y = |b + a\cos 2x|$ for the interval $0 \le x \le 2\pi$. On the same axes, sketch the graph of $y = \dfrac{x}{\pi} + 1$. Hence, state the number of roots of the equation $\pi|b + a\cos 2x| = x + \pi$ in this interval. [5]

5. It is given that $\cos\theta = \dfrac{p^2 - q^2}{p^2 + q^2}$ where $q > p > 0$ and $\tan\theta < 0$. Find $\sin\theta$ in terms of p and q. [3]

6. (a) The diagram shows a garden in the form of a sector of a circle, centre O, radius R m and angle $\dfrac{\pi}{3}$. Within this garden, a circular plot of the largest possible size is to be planted with roses. Given that the radius of this plot is r m,

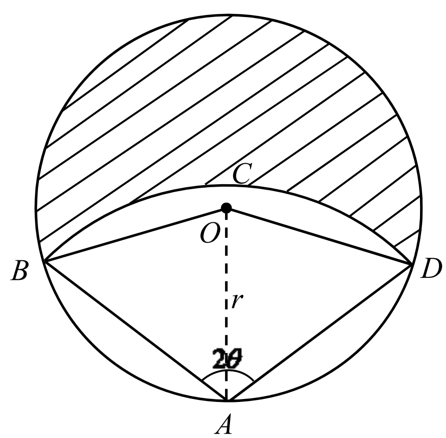


(i) show that $R = 3r$, [2]

(ii) calculate the fraction of the garden that is to be planted with roses. [2]

(b) Find the perimeter of the shaded region. Leave your answer in the form of $r(a + b\sqrt{3})$ m, where a and b are constants. [3]

7. The diagram shows a circle, centre O and radius r cm. $\angle BAD = 2\theta$ radians and OA bisects $\angle BAD$. BCD is an arc with centre A and radius AB. Show that the area of the shaded region is $r^2(\sin 2\theta - 2\theta\cos 2\theta)$ cm^2. [5]



Bonus Question

Given that $\sin\theta$ and $\cos\theta$ are two roots to the equation $2x^2 + (\sqrt{2} + 1)x + 5m = 0$, find the value of m and the value of $\dfrac{\sin\theta}{1 - \cot\theta} + \dfrac{\cos\theta}{1 - \tan\theta}$. [4]

END OF PAPER