

Name: \_\_\_\_\_ ( ) Class: \_\_\_\_\_ Date: 28 Jan 2015

**Circular Measure and Trigonometry**  
**1 hour**

**Class Test 1 –V2**  
**40 marks**

**Instructions:**

- Answer all the questions.
- Omission of essential working will result in loss of marks.
- You are reminded of the need for clear presentation in your answers.
- If the numerical answer is not exact, give your answer to 1 decimal place for angles in degrees and 3 significant figures for all others unless otherwise stated in the question.
- The number of marks is given in brackets [ ] at the end of each question or part question.

1. Without using a calculator, evaluate  $\frac{2 \sin 60^\circ - \tan 30^\circ}{\cos 135^\circ \operatorname{cosec} 45^\circ + \cot 60^\circ}$ . Show your working clearly. [4]

2. Find all angles which satisfy the equations

(a)  $\sin(2x + 40^\circ) = 3 \cos(50^\circ - 2x) - 1$ , for  $0 \leq x \leq 360^\circ$ , [4]

(b)  $\sqrt{3} \sin^2 \theta - 2 \sin \theta \cos \theta - \sqrt{3} \cos^2 \theta = 0$ , for  $0 \leq \theta \leq 2\pi$ . [4]

3. Prove that  $(1 + \sin x - \cos x)^2 \equiv 2(1 + \sin x)(1 - \cos x)$ . [3]

Hence

(i) prove that  $(1 - \sin x - \cos x)^2 \equiv 2(1 - \sin x)(1 - \cos x)$ . [1]

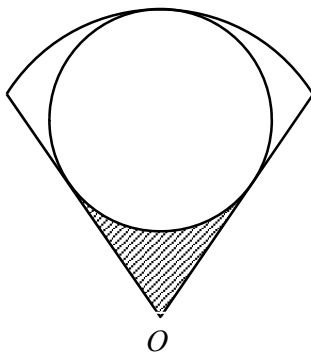
(ii) find all values of  $x$  between 0 and  $2\pi$  inclusive for which  $(1 + \sin x - \cos x)^2 = 1 - \cos x$ . [4]

4. (i) It is given that the function  $f(x) = a \sin x + 2b$ , where  $0 \leq x \leq 2\pi$  and  $a > 0$ ,  $b > 0$  has a maximum value of 5 and a minimum value of -1, find the value of  $a$  and of  $b$ .

(ii) Using the values of  $a$  and  $b$  found in (i), sketch the graph of  $y = |b + a \cos 2x|$  for the interval  $0 \leq x \leq 2\pi$ . On the same axes, sketch the graph of  $y = \frac{x}{\pi} + 1$ . Hence, state the number of roots of the equation  $\pi |b + a \cos 2x| = x + \pi$  in this interval. [5]

5. It is given that  $\cos \theta = \frac{p^2 - q^2}{p^2 + q^2}$  where  $q > p > 0$  and  $\tan \theta < 0$ . Find  $\sin \theta$  in terms of  $p$  and  $q$ . [3]

6. (a) The diagram shows a garden in the form of a sector of a circle, centre  $O$ , radius  $R$  m and angle  $\frac{\pi}{3}$ . Within this garden, a circular plot of the largest possible size is to be planted with roses. Given that the radius of this plot is  $r$  m,

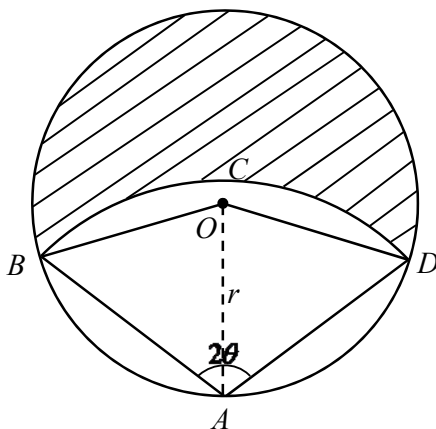


(i) show that  $R = 3r$ , [2]

(ii) calculate the fraction of the garden that is to be planted with roses. [2]

(b) Find the perimeter of the shaded region. Leave your answer in the form of  $r(a + b\sqrt{3})$  m, where  $a$  and  $b$  are constants. [3]

7. The diagram shows a circle, centre  $O$  and radius  $r$  cm.  $\angle BAD = 2\theta$  radians and  $OA$  bisects  $\angle BAD$ .  $BCD$  is an arc with centre  $A$  and radius  $AB$ . Show that the area of the shaded region is  $r^2(\sin 2\theta - 2\theta \cos 2\theta)$  cm<sup>2</sup>. [5]



**Bonus Question**

Given that  $\sin \theta$  and  $\cos \theta$  are two roots to the equation  $2x^2 + (\sqrt{2} + 1)x + 5m = 0$ , find the value of  $m$  and the value of  $\frac{\sin \theta}{1 - \cot \theta} + \frac{\cos \theta}{1 - \tan \theta}$ . [4]