Name: ______________________________ (   )   Class: _________   Date: ***27 Jan 2015***

Circular Measure and Trigonometry **Class Test 1 -V1**
1 hour **40 marks**

Instructions:

- Answer all the questions.
- Omission of essential working will result in loss of marks.
- You are reminded of the need for clear presentation in your answers.
- If the numerical answer is not exact, give your answer to 1 decimal place for angles in degrees and 3 significant figures for all others unless otherwise stated in the question.
- The number of marks is given in brackets [   ] at the end of each question or part question.

1. Find all angles which satisfy the equations

   (a) $\cos(2x - 60°) = \sin 30°$, for $0 \le x \le 360°$, [4]

   (b) $2\cot\theta - \sqrt{3}\tan\theta = 2 - \sqrt{3}$, for $0 \le \theta \le 2\pi$. [4]

2. (i) Prove that $\dfrac{1 - \tan^2 2x}{\sec^2 2x + 2\tan 2x} \equiv \dfrac{\cos 2x - \sin 2x}{\cos 2x + \sin 2x}$. [4]

   (ii) Hence, find all values of *x* between 0 and 4 for which $\dfrac{1 - \tan^2 2x}{\sec^2 2x + 2\tan 2x} = -\dfrac{3}{2}$. [4]

3. (i) It is given that the function $f(x) = b\cos x + a$, where $0 \le x \le 2\pi$ and $a > 0, b < 0$ has a maximum value of 6 and a minimum value of 2, find the value of *a* and of *b*. [1]

   (ii) Using the values of *a* and *b* found in (i), sketch the graph of $y = |a\sin x + b|$ for the interval $0 \le x \le 2\pi$. On the same axes, sketch the graph of $y = \dfrac{2x}{\pi} + \dfrac{1}{2}$. Hence, state the number of roots of the equation $|a\sin x + b| - \dfrac{1}{2} = \dfrac{2x}{\pi}$ in this interval. [4]

4. It is given that $\dfrac{\sin^2\beta}{5\cos^2\beta - 1} = \dfrac{4}{5}$, where $90° \le \beta < 180°$. Without using a calculator, find the value of $\dfrac{\sin\beta}{5\cos\beta - 1}$. [4]

5. The centres A, B and C of three identical circles, of radius 2 cm, are the vertices of an equilateral triangle of side 4 cm. Each side of the equilateral triangle PQR touches two of the arcs as shown, where X and Y are the points of contact.

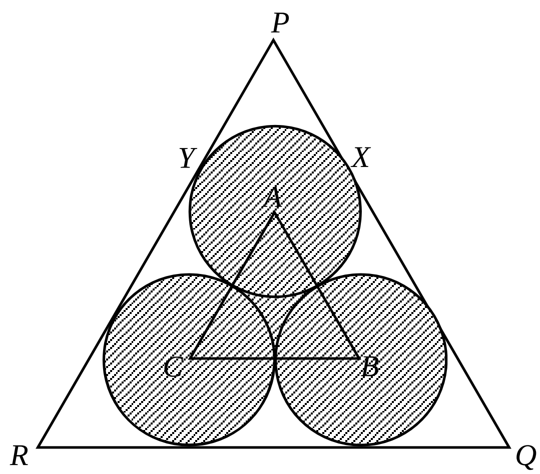
Show that PX is $2\sqrt{3}$ cm. [2]

Hence,

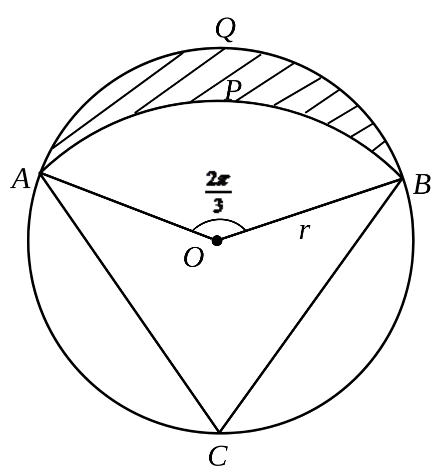
(i) calculate the perimeter of the unshaded region PXY. [2]

(ii) express the area of triangle PQR in the form $a+b\sqrt{3}$, where a and b are constants. [3]

(iii) find the total area of the unshaded regions. [3]



6. The diagram shows a circle $AQBC$, centre O, radius r cm. An arc APB is drawn with centre C on the circle. Given that $\angle AOB$ is $\frac{2\pi}{3}$ radians, show that the area of the shaded region is $r^2\left(\frac{\sqrt{3}}{2}-\frac{\pi}{6}\right)$ cm^2. [5]



Bonus Question

Given that $\sin x+\cos x=\frac{\sqrt{2}}{3}$, where $270° \le x \le 360°$, evaluate $\sin x-\cos x$ and hence show that $\tan x=\frac{\sqrt{2}-4}{\sqrt{2}+4}$. [4]

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