

Name: _____ () Class: _____ Date: 27 Jan 2015

Circular Measure and Trigonometry
1 hour

Class Test 1 -V1
40 marks

Instructions:

- Answer all the questions.
- Omission of essential working will result in loss of marks.
- You are reminded of the need for clear presentation in your answers.
- If the numerical answer is not exact, give your answer to 1 decimal place for angles in degrees and 3 significant figures for all others unless otherwise stated in the question.
- The number of marks is given in brackets [] at the end of each question or part question.

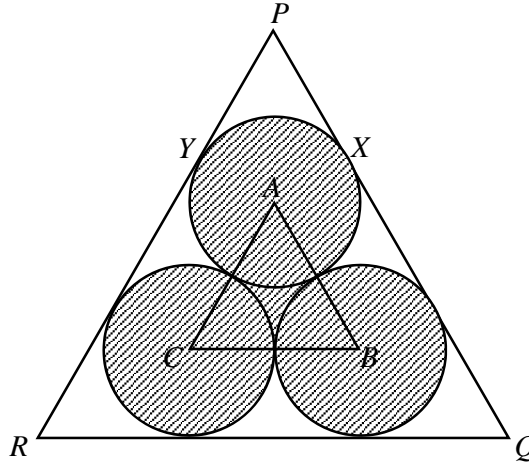
- Find all angles which satisfy the equations
 - $\cos(2x - 60^\circ) = \sin 30^\circ$, for $0 \leq x \leq 360^\circ$, [4]
 - $2 \cot \theta - \sqrt{3} \tan \theta = 2 - \sqrt{3}$, for $0 \leq \theta \leq 2\pi$. [4]
- Prove that $\frac{1 - \tan^2 2x}{\sec^2 2x + 2 \tan 2x} \equiv \frac{\cos 2x - \sin 2x}{\cos 2x + \sin 2x}$. [4]
 - Hence, find all values of x between 0 and 4 for which $\frac{1 - \tan^2 2x}{\sec^2 2x + 2 \tan 2x} = -\frac{3}{2}$. [4]
- It is given that the function $f(x) = b \cos x + a$, where $0 \leq x \leq 2\pi$ and $a > 0, b < 0$ has a maximum value of 6 and a minimum value of 2, find the value of a and of b . [1]
 - Using the values of a and b found in (i), sketch the graph of $y = |a \sin x + b|$ for the interval $0 \leq x \leq 2\pi$. On the same axes, sketch the graph of $y = \frac{2x}{\pi} + \frac{1}{2}$. Hence, state the number of roots of the equation $|a \sin x + b| - \frac{1}{2} = \frac{2x}{\pi}$ in this interval. [4]
- It is given that $\frac{\sin^2 \beta}{5 \cos^2 \beta - 1} = \frac{4}{5}$, where $90^\circ \leq \beta < 180^\circ$. Without using a calculator, find the value of $\frac{\sin \beta}{5 \cos \beta - 1}$. [4]

5. The centres A , B and C of three identical circles, of radius 2 cm, are the vertices of an equilateral triangle of side 4 cm. Each side of the equilateral triangle PQR touches two of the arcs as shown, where X and Y are the points of contact.

Show that PX is $2\sqrt{3}$ cm. [2]

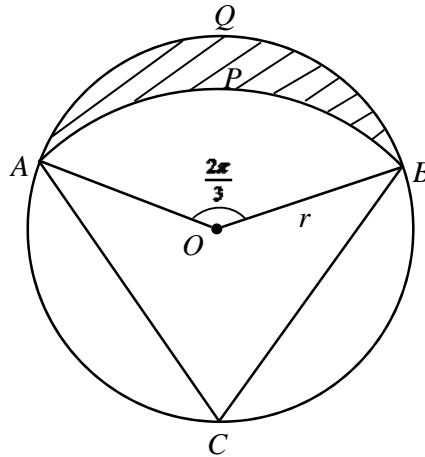
Hence,

- (i) calculate the perimeter of the unshaded region PXY . [2]
 (ii) express the area of triangle PQR in the form $a + b\sqrt{3}$, where a and b are constants. [3]
 (iii) find the total area of the unshaded regions. [3]



6. The diagram shows a circle $AQBC$, centre O , radius r cm. An arc APB is drawn with centre C on the circle. Given that $\angle AOB$ is $\frac{2\pi}{3}$ radians, show that the area of the shaded region is

$$r^2 \left(\frac{\sqrt{3}}{2} - \frac{\pi}{6} \right) \text{cm}^2. \quad [5]$$



Bonus Question

Given that $\sin x + \cos x = \frac{\sqrt{2}}{3}$, where $270^\circ \leq x \leq 360^\circ$, evaluate $\sin x - \cos x$ and hence show that

$$\tan x = \frac{\sqrt{2} - 4}{\sqrt{2} + 4}. \quad [4]$$

END OF PAPER