REVISION (END OF YEAR) 2015

Algebraic Manipulation

1. Factorize the following expressions completely.

(a) $(a+2b)^4 - 2(a+2b)^2 - 63$ [2]

(b) $(x+6)(x+9)+(x+7)(x+8)+(x+6)(x+8)+(x+7)(x+9)$ [3]

2. Given that $\frac{a}{2d} = \frac{1}{b}\sqrt{\frac{c^2-1}{2-c^2}}$, make c the subject of the formula. [4]

3. Find, in its simplest form, the product of $(x^{\frac{2}{3}} - x^{\frac{1}{3}}y^{\frac{2}{3}} + y^{\frac{4}{3}})$ and $(x^{\frac{1}{3}} + y^{\frac{2}{3}})$. [2]

4. Solve the following simultaneous equations [3]

$$\frac{4^x}{8^{y+1}} = 1$$
$$9^x = 9 \times 27^{2y}$$

5. Solve the following equations

(a) $8^{x+1} - 4^{-x}2^{-x-3} = 0$ [3]

(b) $\frac{5}{2x-1} + \frac{3}{2-4x} - \frac{2}{6x-3} = 1$ [3]

Surds

6. (a) ΔABC is such that AB is $(2+\sqrt{8})$ m and it area is given by $\left(\frac{1}{2}+\frac{3}{2}\sqrt{2}\right)$m^2.

Express the perpendicular distance from C to AB in the form of $(m+n\sqrt{2})$ where m and n are real numbers. [3]

(b) Express $\left(11+\sqrt{3}\right) - \left(\frac{13}{4+\sqrt{3}}\right)^2$ in the form of $a+b\sqrt{3}$, where a and b are integers. [3]

7. Solve the equations

(a) $\sqrt{4-3x} = \dfrac{x+12}{2}$, [3]

(b) $\sqrt{4+x^2-\sqrt{2x-1}} = 1+x$. [4]

8. (a) Simplify $\sqrt{\sqrt{2}+\sqrt{2-\sqrt{2}}} \times \sqrt{\sqrt{2}-\sqrt{2-\sqrt{2}}}$. [2]

Quadratic Equations

9. By expressing $-3x^2+21x-18$ in the form $a(x-b)^2+c$, where a, b and c are constants.

(i) Write down the maximum value of $-3x^2+21x-18$. [2]

(ii) Hence, sketch the graph of $-3x^2+21x-18$, indicating clearly all workings, intercepts and turning point. [3]

10. Show that for all real values of p and q, $y = -(1+p^2)x^2 + 2pqx - (2q^2+1)$ is always negative for all real values of x. [4]

11. Given a, b and c $(a \neq c)$ to be the three sides of a scalene triangle ABC. Find the nature of the root(s) of the quadratic equation $(a-c)x^2+2bx-(c-a)=0$. Show all your workings and reasoning(s) clearly. [4]

12. Find the range of values of w for which $-20 < \left(\sqrt{w}-5\right)\left(w+4\right) < 0$. [4]

13. Show that the equation $\dfrac{2x+3}{x-1} + \dfrac{3x-4}{x+1} = 2$ has no real solutions. [4]

Functions

14. Given that $\mathrm{f}^{-1} : x \mapsto \dfrac{x+\sqrt{x^2-4}}{2}$, $x \geq k$.

(i) State the value of k. [1]

(ii) Find f(3). [2]

From HCI Resources

15. (a) Sketch in the same diagram. Indicate the key points clearly. [6]

(i) $y = \frac{3}{x^2}$ (ii) $y^2 = x + 1$ (iii) $y = 3x^{\frac{1}{2}}$ (iv) $y = \ln(x+1)^2$

(b) Describe **two successive geometrical transformations** on how you would obtain the following graphs from the graph of $y = \mathrm{f}(x)$. **You are not required to draw them.**

(i) $y = |\mathrm{f}(x)| + 1$ [2]

(ii) $y = \frac{1}{2}\mathrm{f}(-x)$ [2]

<u>Polynomials</u>

16. (a) Given that $\mathrm{f}(x) = x^3 - mx^2 + nx - 32$, find the value of *m* and of *n* if f(*x*) has a factor $(x-1)$ but leaves a remainder of 10 when divided by $(x-3)$. [3]

(b) Given that *a*, *b*, *c* and *d* are all positive integers. Prove that $x^{4a} + x^{4b+1} + x^{4c+2} + x^{4d+3}$ has a factor of $x^3 + x^2 + x + 1$. [2]

17. The expressions $x^3 - x^2 - 3x + 2$ and $2x^3 - 7x^2 + 12$ have a common factor $x - 2$.
Explain why the expression $\mathrm{f}(x) = x^3 - 6x^2 + 3x + 10$ also has $x - 2$ as a factor. [2]

(i) Hence or otherwise, solve completely the equation $x^3 - 6x^2 + 3x + 10 = 0$. [4]

(ii) Sketch the graph of $y = \mathrm{f}(x)$, labeled all *x* and *y* intercepts clearly. [2]

(iii) Hence, find the range of values of *x* for which $\frac{\mathrm{f}(x)}{x^2} \geq 0$. [2]

18. Find the value of k for which $x^2 + kx + 8$ is a factor of $x^3 - 5x^2 + 2x + 8$. Hence, or otherwise, solve the equation $8(8^{x-1} + 1) + 2^{x+1} = 5(4^x)$. [6]

Variation

19. The period P of oscillation of a simple pendulum varies as the square root of its length l. Then the length is 16 cm, the period is 1.2 seconds. Find

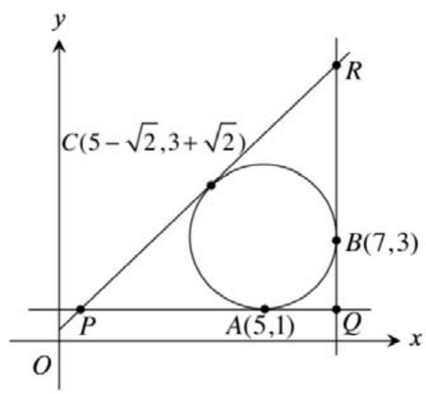
(i) the period when the length is 64 cm. [2]

(ii) the length when the period is 3 seconds. [2]

Coordinates Geometry + Circles

20. The diagram shows a circle whose equation is given by $(x-5)^2+(y-3)^2=4$ and

three tangents of the circle at $A(5, 1)$, $B(7, 3)$ and $C\left(5-\sqrt{2},\ 3+\sqrt{2}\right)$. The tangents intersect at points P, Q and R as shown. Tangent PQ is parallel to the x-axis and tangent QR is parallel to the y-axis.



Find

(i) the equations of the tangents PR, PQ and QR, [4]

(ii) the coordinates of P, Q and R, [4]

(iii) the area of ΔPQR. [2]

21. If three points A (2, 2), B (a, 0) and C (0, b) are collinear ($ab \neq 0$), find the value of $\frac{1}{a}+\frac{1}{b}$. [2]

Logarithmic

22. (a) Show that $\ln\left(\frac{1}{x}\right) = \log_{\frac{1}{e}} x$. [2]

(b) Sketch the graph of $y = \ln\left(\frac{1}{x}\right)$. [1]

(c) In the same diagram, draw a suitable graph to find the number of solutions in the equation $-5(x+3) = \dfrac{1}{\ln\left(\frac{e^2}{x}\right)^5}$. [4]

Matrices

23. Two outlets sold the following beverages on a particular day,

	Tea	Coffee	Milo
Orchard outlet	35	73	58
Clementi outlet	28	52	47

It costs \$0.50, \$0.80 and \$0.30 for the outlets to prepare a cup of Tea, Coffee and Milo respectively. Each cup of Tea, Coffee and Milo was sold for \$3.50, \$4.80 and \$2.50 respectively.

Given that $Q = \begin{pmatrix} 35 & 73 & 58 \\ 28 & 52 & 47 \end{pmatrix}$, $C = \begin{pmatrix} 0.50 \\ 0.80 \\ 0.30 \end{pmatrix}$ and $S = \begin{pmatrix} 3.50 \\ 4.80 \\ 2.50 \end{pmatrix}$,

(i) find $S - C$. [1]

(ii) find $Q(S - C)$ and interpret the results. [3]

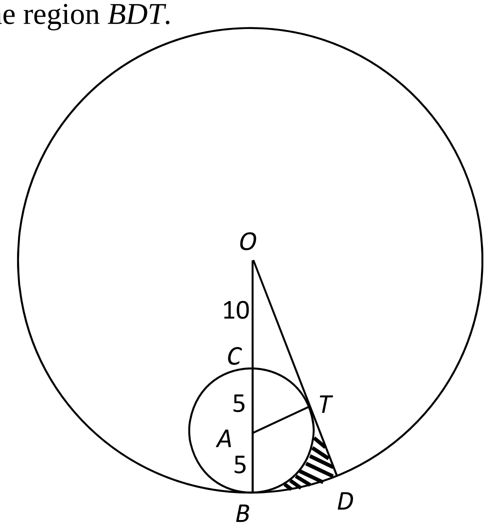
24. Given that $a > b > 0$ and $\begin{pmatrix} a & b \\ a & b \end{pmatrix}\begin{pmatrix} a & b \\ b & a \end{pmatrix} = \begin{pmatrix} 5 & 4 \\ m & n \end{pmatrix}$, write down the values of *m* and *n*. Find the values of *a* and *b*. [6]

Trigonometry

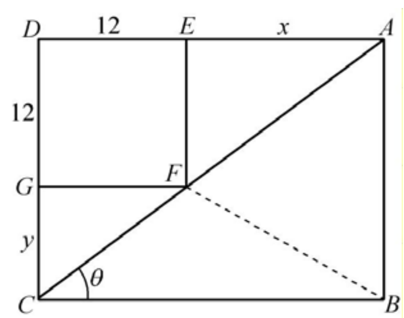
25. The figure below shows two circles with centres O and A respectively. The circles touch each other internally at B. OD is the tangent to the smaller circle at T. If $AB = 5$ cm and $OC = CB$, find

(i) $\angle AOT$ and $\angle BAT$ in radians, [3]

(ii) the shaded area of the region BDT. [3]



26. In the diagram, $ABCD$ is a rectangle, E, F and G are the points on AD, AC and DC respectively such that $DEFG$ is a square of side 12 cm. Given that $\sin\theta = \frac{3}{5}$, $\cos\ \theta = \frac{4}{5}$ and $\tan\ \theta = \frac{3}{4}$ where $\angle ACB = \theta$. Without using a calculator, show that $FB^2 = 337$. [4]



27. The curve of equation $y = a\cos(x - 30°)$, $0° \le x \le 360°$, where a is a constant. The curve meets the y-axis at $(0°, \sqrt{3})$ and passes through the points $(p, 0)$ and $(q, 0)$.

(i) Find the value of a. [2]

(ii) State the value of p and of q. [2]

From HCI Resources

28. P, Q, R and S are 4 points on the ground. The bearing of R from S is 115°. The bearing of Q from R is 215°. The bearing of P from R is 240°.

(a) Given that $PQ = QR = 50$ m, find the length of PR [3]

(b) Find the bearing of P from S if PS is 95 m. [3]

(c) Calculate the distance RS. [2]

(d) Calculate the distance QS. [2]

29. (a) Find all the angles x, where $0^\circ \le x \le 360^\circ$, that satisfy the following equation

$$1 + 5\sin\left(\frac{x}{3} + 120^\circ\right) = 0.$$ [3]

(b) If $\cos x = -\frac{4}{5}$ and $\pi < x < \frac{3}{2}\pi$, calculate without the use of calculator, the exact value of

(i) $\tan x$,

(ii) $\sin(\frac{\pi}{2} + x)$,

(iii) $\cos 2x$. [6]

30. Prove the identity: $\dfrac{\tan^2\theta}{(1+\tan^2\theta)^4} + \dfrac{\cot^2\theta}{(1+\cot^2\theta)^4} \equiv \frac{1}{8}\sin^2 2\theta(2 - \sin^2 2\theta)$ [3]

31. Sketch on the same diagram the graphs of $y = 3\cos(2x) + 2$ and $y = 1 - |\sin x|$ for the domain $0° \le x \le 360°$. Hence, state the number of solutions in the equations

(i) $3\cos(2x) + 3 = \dfrac{2x}{\pi}$,

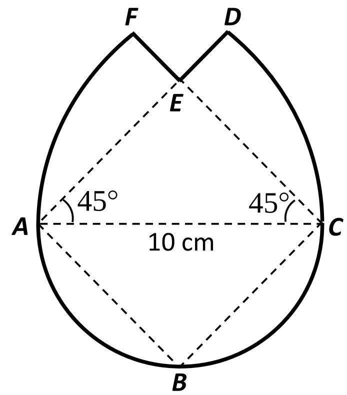
(ii) $3\cos(2x) + |\sin x| + 1 = 0$. [6]

Circular Measure

32. The diagram shows a planar figure which is formed by overlapping two identical sectors ACD and CAF, and placed above a semicircle whose diameter is AC. Given that $AC = 10$ cm, find the

(a) area, and [5]

(b) perimeter of the figure. [4]



From HCI Resources

Binomial Theorem

33. (a) Given that the coefficient of x^4 is $\frac{35}{16}$ in the expansion of $(3+px)\left(1-\frac{x}{2}\right)^7$, find the value of p. [5]

(b) Prove that $2^n\left[\binom{2}{2}\binom{4}{2}\binom{6}{2}\cdots\binom{2n-2}{2}\binom{2n}{2}\right]\equiv(2n)!$, $n\in Z^+$. [2]

Further Trigonometry

34. Show that $\sin 2x+\sin 4x+\sin 6x=4\cos x\cos 2x\sin 3x$. [4]

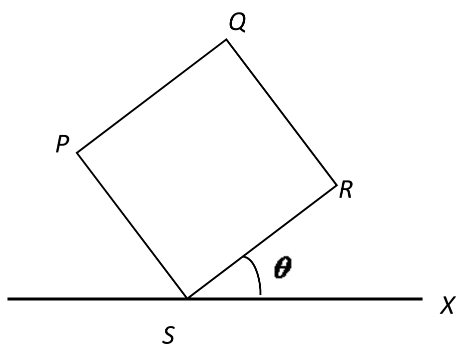
Hence evaluate the value of $\cos 15°$, leaving your answers in surd form. [3]

35. In the diagram, *PQRS* is a square of side $\sqrt{2}$ cm, *RS* makes an angle of $\theta°$ with the horizontal *SX*. Let *h* cm be the perpendicular distance from *Q* to *SX*.

(i) Show that $h=\sqrt{2}\left(\cos\theta+\sin\theta\right)$, [2]

(ii) Express *h* in the form $R\sin(\theta+\alpha)$, [2]

(iii) Find the maximum value of *h* and the corresponding value of θ. [2]



36. It is given that $\cos 3x=4\cos^3 x-3\cos x$. Evaluate, without the use of calculators, the value of

$$\frac{3}{\sin^2 20°}-\frac{1}{\cos^2 20°}+64\sin^2 20°$$ [4]

37. Prove that for all real values of *x*, $\sin x+\sin(60°-x)=\sin(60°+x)$. Hence, solve the equation for $0°\le x\le 360°$, $4\sin 2x+\sin(60°-2x)=\sin(60°+2x)-2$. [6]

Differentiation

38. If $6y+9x^2+60x=2x^3+30$, find the range of values of *x* for which $\frac{dy}{dx}<0$. [4]

From HCI Resources

39. The equation of a curve is defined by $\frac{1}{\sqrt{x}}-\frac{1}{\sqrt{y}}=\frac{1}{6}$, where $x > 0$ and $y > 0$.

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ and the equation of the tangent to the curve at $(4,9)$. [4]

40. A curve is given as $y=\frac{x^2+x+2}{x-1}$.

(i) By finding $\frac{\mathrm{d}y}{\mathrm{d}x}$ and $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$, determine the nature of its stationary points. [5]

(ii) By expressing $y=\frac{x^2+x+2}{x-1}$ in the form of $Ax+B+\frac{C}{x-1}$, find the equation of its asymptote(s). [3]

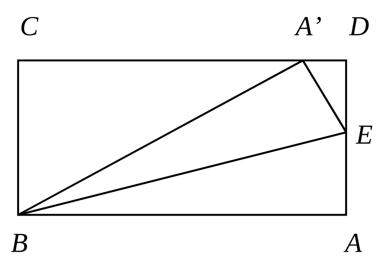
(iii) Hence sketch the curve $y=\frac{x^2+x+2}{x-1}$. [2]

41. (a) Given that $\frac{\mathrm{d}}{\mathrm{d}x}(1+x)^n=n(1+x)^{n-1}$, show that $\binom{n}{1}+2\binom{n}{2}+3\binom{n}{3}+...+n\binom{n}{n}=n\times 2^{n-1}$. [4]

(b) Given that $\mathrm{f}(x)=\cos x+\sin x$, find $\mathrm{f}'(x), \mathrm{f}''(x), \mathrm{f}'''(x)$ and $\mathrm{f}^4(x)$. Hence find

$\mathrm{f}'(\frac{\pi}{2})+\mathrm{f}''(\frac{\pi}{2})+\mathrm{f}'''(\frac{\pi}{2})+...+\mathrm{f}^{2014}(\frac{\pi}{2})$. [4]

42. The figure below shows a piece of rectangular paper. *ABCD* with its corner, *A* folded along line *BE* to a point A' on *CD*. Let *AD* = 3 cm, *AB* = *y* cm and *AE* = *x* cm.



(i) Show that $A'D=\sqrt{6x-9}$. [2]

(ii) By finding *y*, prove that the area of triangle $A'BE=\frac{3x^2}{2\sqrt{6x-9}}$. [3]

(iii) Find the minimum area and the corresponding value of *x*. [4]

From HCI Resources

43. A family of curves is given by $y = Ae^{x} + 2\sin x + B$ where A and B are arbitrary constants. Find the value of k such that $\frac{\mathrm{d}y}{\mathrm{d}x} - \frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = k(\sin x + \cos x)$. [4]

Integration

44. (a) Integrate with respect to x.

(i) $\int 3\mathrm{e}^{\tan x} \sec^2 x \, \mathrm{d}x$, [2]

(ii) $\int \frac{1}{1+\mathrm{e}x} \, \mathrm{d}x$, [2]

(b) Evaluate

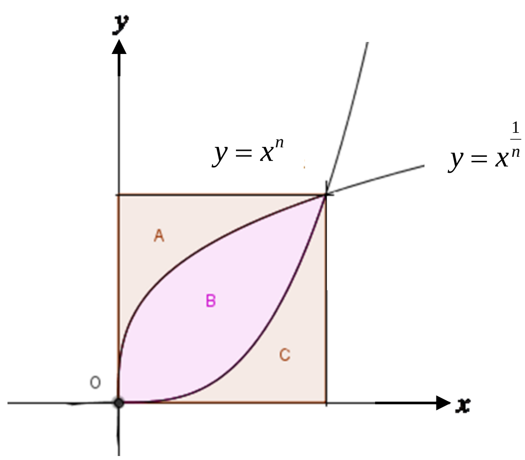
(i) $\int_4^5 \frac{x^3 + 4x^2 + 3x}{x^2 - 9} \, \mathrm{d}x$, [4]

(ii) $\int_0^{\frac{\pi}{32}} \frac{8}{1-\sin 8x} + \frac{8}{1+\sin 8x} \, \mathrm{d}x$. [4]

45. (i) Given that $\frac{\mathrm{d}}{\mathrm{d}x}\left[2\mathrm{e}^{x}\left(\sin x - \cos x\right)\right] = a\mathrm{e}^{x} \sin x$, find the value of a. [2]

(ii) Hence, evaluate $\int_0^{\pi} \mathrm{e}^{x} \sin x \, \mathrm{d}x$ expressing the value in terms of e. [3]

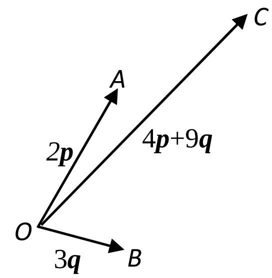
46. Find the value of n such that the area of A + C = area of B. [4]



From HCI Resources

Vectors

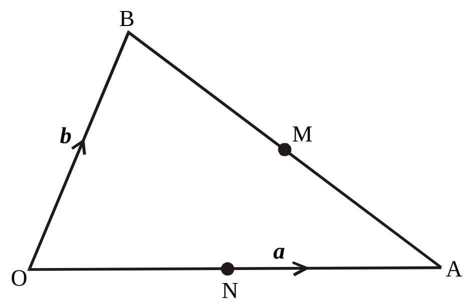
47. In the diagram, $\overrightarrow{OA} = 2\mathbf{p}, \overrightarrow{OB} = 3\mathbf{q}$ and $\overrightarrow{OC} = 4\mathbf{p} + 9\mathbf{q}$.



(i) Given that the point P is such that $\overrightarrow{AP} = 2\overrightarrow{PB}$, find the position vector of P in terms of $\boldsymbol{p}$ and $\boldsymbol{q}$. [3]

(ii) Given that point Q is such that $\overrightarrow{OQ} = 3\overrightarrow{OP}$, express $\overrightarrow{OQ}$ in terms of $\boldsymbol{p}$ and $\boldsymbol{q}$. Show that Q lies on BC and write down the numerical value of $\dfrac{BQ}{QC}$. [3]

48.



In the triangle OAB, M is the midpoint of AB and N is the midpoint of OA. Given that $\overrightarrow{OA} = \boldsymbol{a}$ and $\overrightarrow{OB} = \boldsymbol{b}$, express in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$,

(i) $\overrightarrow{AB}$, [1]

(ii) $\overrightarrow{AM}$, [1]

(iii) $\overrightarrow{OM}$, [1]

(iv) $\overrightarrow{BN}$. [1]

P lies on OM such that $OP = \dfrac{2}{3}OM$.

(v) Express $\overrightarrow{BP}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$. [2]

(vi) Express $\overrightarrow{BP}$ in terms of $\overrightarrow{BN}$. Explain the geometrical significance of this relationship. [2]

(vii) Find the ratio $\dfrac{\text{Area of } \Delta BPM}{\text{Area of } \Delta OPN}$ [2]

(viii) Find the ratio $\dfrac{\text{Area of } \Delta BPM}{\text{Area of quadrilateral } PNAM}$ [2]

From HCI Resources

Probability

49. A sample of 3 switches is taken from a box containing 8. Three switches in the box are defective. What is the probability that a sample has

(a) no defective switches [1]

(b) at least two defective switches [2]

(c) at least one good switch [2]

50. 8 girls and 12 boys, including John himself was invited to take part in a lucky draw. The organiser will draw three winners from the group.

(i) What is the probability of getting all winners with the same gender if each winner is only entitled to one prize? [2]

(ii) Find the probability of John winning the first prize only. [2]

(iii) What is the probability of John winning all the three prizes? [2]

ALL THE BEST

Answers

1a. $[(a+2b)^2+7][(a+2b)^2-9]$ 1b. $(2x+13)(2x+17)$ 2. $c=\pm 1$ 3. $x+y^2$ 4. $x=2, y=\dfrac{1}{3}$

5a. $x=-1$ 5b. $x=\dfrac{23}{12}$ 6a. $-\dfrac{5}{4}+\dfrac{1}{2}\sqrt{2}$ m b. $30-7\sqrt{3}$ 7a. $x=-4$ 7b. $x=1$ 8. $\sqrt[4]{2}$ 9i. $18\dfrac{3}{4}$

12. $0<w<1$ or $16<w<25$ 13. Discriminant<0 14i. $k=2$ 14ii. $3\dfrac{1}{3}$ 16a. $m=13, n=44$

17ii. $f(x)=(x-2)(x-5)(x+1)$ 17iii. $-1\le x<0$ or $0<x\le 2$ or $x\ge 5$ 18. $k=6$; $x=1$ or $x=2$ 19i. 2.4s

19ii. 100cm 20i. $y=x-2+2\sqrt{2}; y=1; x=7$ 20ii. $P=(3-2\sqrt{2},1), Q=(7,1), R=(7,5+2\sqrt{2})$

20iii. $(12+8\sqrt{2})$sq units 21. $\dfrac{1}{2}$ 22c. 1 solution 23i. $\begin{pmatrix}3\\4\\2.2\end{pmatrix}$

23ii. $\begin{pmatrix}524.6\\395.4\end{pmatrix}$; total profits from each outlet 24. $m=5, n=4$; $a=2, b=1$ or $a=1, b=2$ 25i. 0.340;1.91

25ii. 8.73cm^2 27i. 2 27ii. $p=120°, q=300°$ 28a. 90.6m 28b. 166.4° 28c.111m 28d.114m

29a. 214.6° 29bi. $\dfrac{3}{4}$ 29bii. $-\dfrac{4}{5}$ 29biii. $\dfrac{7}{25}$ 31i.4 solutions 31ii.4 solutions 32a. $(75\pi-25)\text{cm}^2$

32b.53.0cm 33a. $p=1$ 34. $\dfrac{\sqrt{6}+\sqrt{2}}{4}$ 35i. $h=2\sin(\theta+45°)$ 35iii. 2cm; $\theta=45°$ 36.32

37. 110.9°,159.1°,290.9°,339.1° 38. $-2<x<5$ 39. $y=\dfrac{27}{8}x-\dfrac{9}{2}$

40. $\dfrac{x^2-2x-3}{(x-1)^2}; \dfrac{8}{(x-1)^3}$; $(-1,1)$ is a max. point, $(3,7)$ is a min. point ; $x=1; y=x+2$ 41b. -2

42ii. $y=\dfrac{3x}{\sqrt{6x-9}}$ 42iii. $x=2$ 43. $k=2$ 44ai. $3e^{\tan x}+c$ 44aii. $\dfrac{1}{e}\ln|1+ex|+c$ 44bi. $8\dfrac{1}{2}+12\ln 2$

44bii.2 45i. $a=4$ 45ii. $\dfrac{1}{2}(1+e^{\pi})$ 46. $n=3$ 47i. $\dfrac{2}{3}\mathbf{p}+2\mathbf{q}$ 47ii. $2\mathbf{p}+6\mathbf{q}$; $\dfrac{BQ}{QC}=1$ 48i. $\mathbf{b}-\mathbf{a}$

48ii. $\dfrac{1}{2}\mathbf{b}-\dfrac{1}{2}\mathbf{a}$ 48iii. $\dfrac{1}{2}\mathbf{b}+\dfrac{1}{2}\mathbf{a}$ 48iv. $\dfrac{1}{2}\mathbf{a}-\mathbf{b}$ 48v. $\dfrac{1}{3}\mathbf{a}-\dfrac{2}{3}\mathbf{b}$ 48vi. $\overrightarrow{BP}=\dfrac{2}{3}\overrightarrow{BN}$ 48vii.1 48viii. $\dfrac{1}{2}$

49a. $\dfrac{5}{28}$ 49b. $\dfrac{2}{7}$ 49c. $\dfrac{55}{56}$ 50i. $\dfrac{23}{95}$ 50ii. $\dfrac{361}{8000}$ 50iii. $\dfrac{1}{8000}$