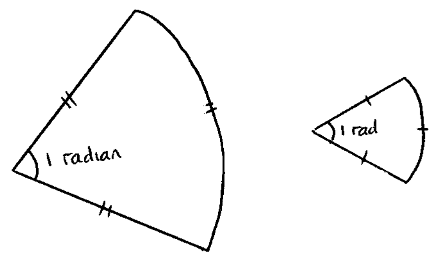
Topic: Circular Measure

Measuring angles in radians

"A radian is the angle subtended at the centre of a circle by an arc length whose length is equal to that of the radius of the circle". This means a radian is the angle formed when the arc length and the radius are the same.

$$\text{No. of Radians in a circle} = \frac{\text{length of circumference}}{\text{radius}}$$
$$= \frac{2\pi r}{r}$$
$$= 2\pi$$



$$\therefore\ 360° = 2\pi\ rads$$
$$180° = \pi\ rads$$
$$1\ rad = \frac{180}{\pi} \approx 57{\cdot}3°$$

Converting angles from Degrees to Radians

1. Convert 45° to radians

$$45° \times \frac{\pi}{180°} = \frac{45\pi}{180}$$
$$= \frac{\pi}{4}$$

← Leaving your answer in terms of π (exact)

2. Convert 75° to radians

$$75° \times \frac{\pi}{180°} = \frac{75\pi}{180}$$
$$= 1.308...$$
$$\approx 1.31$$

← Leaving your answer in 3 sig. fig. (approximated)

Converting angles from Radians to Degrees

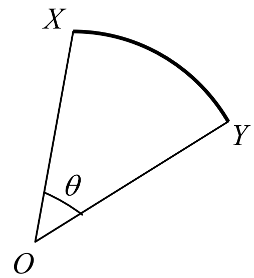
1. Convert $\frac{2\pi}{3}$ rads to degrees

$$\frac{2\pi}{3} \times \frac{180}{\pi} = 120°$$

2. Convert 2.1 rads to degrees

$$2{\cdot}1 \times \frac{180}{\pi} = 120{\cdot}3°$$

<u>Finding Arc Length</u>



When angles are measured in degrees:

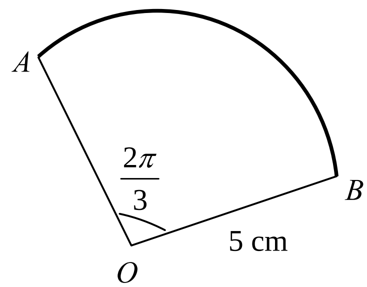
Length of Arc XY, $s = \frac{\theta^\circ}{360^\circ} \times 2\pi r$

When angles are measured in radians:

Length of Arc XY, $s = \frac{\theta}{2\pi} \times 2\pi r$

$= r\theta$

1. Find the length of the arc AB.

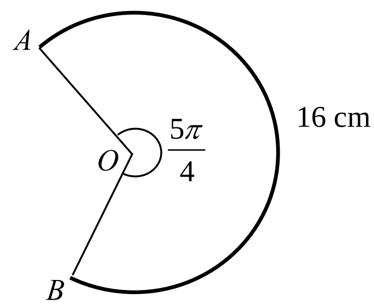


Length of Arc $AB = r\theta$

$= 5\left(\frac{2\pi}{3}\right)$

$= \frac{10\pi}{3}$ cm

2. Find the radius of the sector ABC.



Length of Arc $AB = r\theta$

$16 = r\left(\frac{5\pi}{4}\right)$

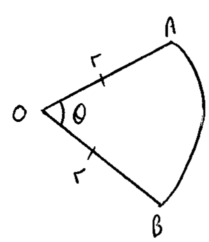
$r = \frac{64}{5\pi}$ cm

3. An arc AB of a circle, centre O and radius r, subtends an angle of θ radians. Given that the perimeter of the sector AOB is P cm, express r in terms of P and θ.

$P = 2r + r\theta$

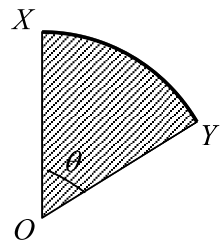
$P = r(2+\theta)$

$r = \frac{P}{2+\theta}$ cm



Finding Area of Sector

When angles are measured in degrees:



Area of Sector OXY, $A = \frac{\theta^\circ}{360^\circ} \times \pi r^2$

When angles are measured in radians:

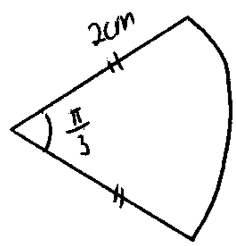
Area of Sector OXY, $A = \frac{\theta}{2\pi} \times \pi r^2$

$= \frac{1}{2} r^2 \theta$

1. Find the area of the sector ABC, where $\angle ABC = \frac{\pi}{3}$ and r = 2 cm.

Area of Sector $ABC = \frac{1}{2}(2)^2\left(\frac{\pi}{3}\right)$

$= \frac{2\pi}{3}$ cm^2

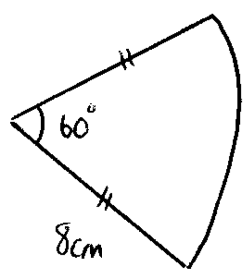


2. Find the area of the sector ABC, where $\angle ABC = 60^\circ$ and r = 8 cm.

Area of Sector $ABC = \frac{1}{2}(8)^2\left(\frac{\pi}{3}\right)$

$= \frac{32\pi}{3}$ cm^2

Remember to convert the angle to radians first!

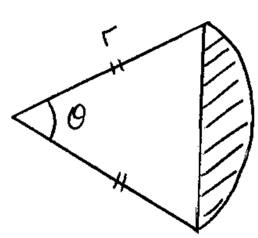


Finding the Area of Segment

Area of segment = Area of sector – area of a triangle

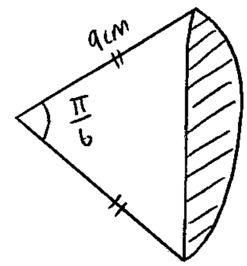
Area of Segment $= \frac{1}{2} r^2 \theta - \frac{1}{2} r^2 \sin\theta$

$= \frac{1}{2} r^2 (\theta - \sin\theta)$



1. Find the area of the shaded segment.

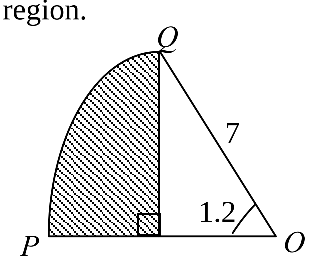
$$\begin{aligned}\text{Area of segment} &= \frac{1}{2}r^2(\theta - \sin\theta)\\ &= \frac{1}{2}(9)^2\left(\frac{\pi}{6} - \sin\frac{\pi}{6}\right)\\ &= 0.95575...\\ &\approx 0.956\ \text{cm}^2\end{aligned}$$



Exercise

For all questions, give all answers for θ in radians.

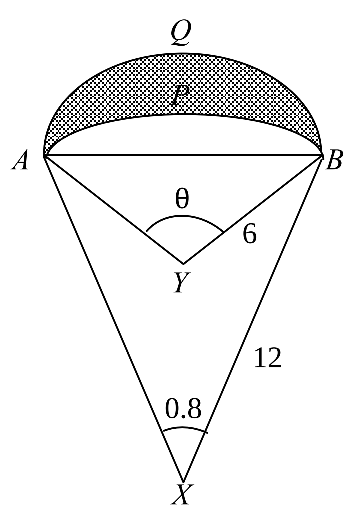
Q1 The diagram shows part of a circle, centre *O* and radius 7 cm. ∠*POQ* is 1.2 radians. Find the area and perimeter of the shaded region.



Q2 In a sketch of a parachute below, *APB* is an arc centre *X*, radius 12 cm and ∠*AXB* = 0.8 radians. *AQB* is an arc centre *Y*, radius 6 cm and ∠*AYB* = θ radians. Calculate the

(i) value of θ,

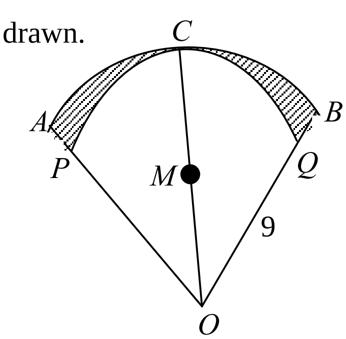
(ii) area of the shaded region.



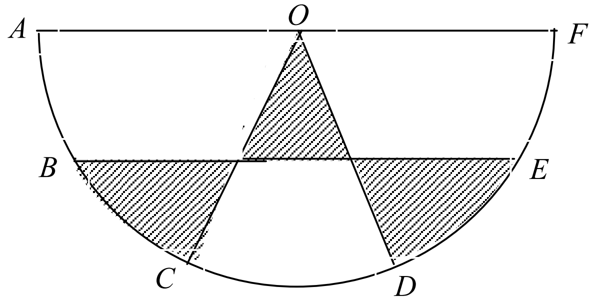
Q3 In the diagram, the sector *OAB* has centre *O*, radius 9 cm and ∠AOB = $\frac{\pi}{6}$ radians. *OC* bisects ∠*AOB* and *M* is the midpoint of *OC*. An arc *PQ* with centre *M* is drawn.

(a) Find ∠*OMP*.

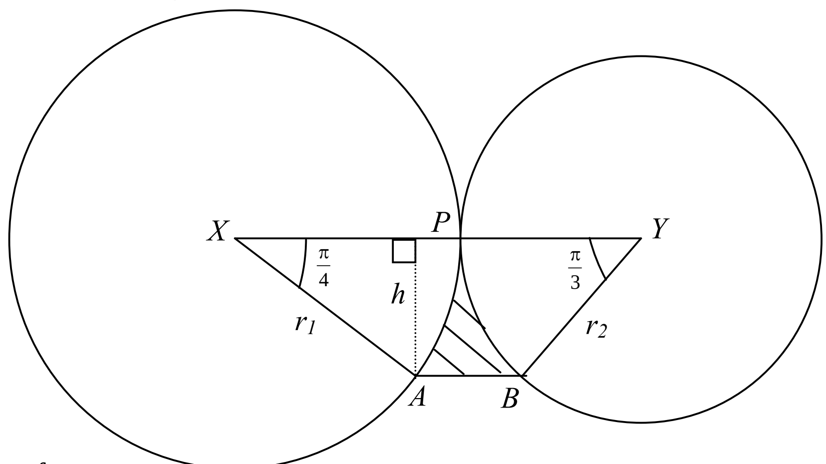
(b) Calculate the perimeter of the shaded region.



Q4 The diagram below shows a semi-circular school field, centre O. During National Day, the uniform groups assemble in the shaded regions of the field. Given that the radius is 30 m, arc AB = arc CD = arc EF = 15 m, find $\angle BOE$ and hence, the perimeter of the shaded region.



Q5 The figure below shows two circles, centres X and Y, radii r_1 cm and r_2 cm which touches externally at P. Given that $\angle AXP = \frac{\pi}{4}$ radians, $\angle BYP = \frac{\pi}{3}$ radians and AB is parallel to XPY,



(a) **(i)** Express h in terms of r_1.
(ii) Express h in terms of r_2.
(iii) Hence, show that $\frac{r_1}{r_2} = \frac{1}{2}\sqrt{6}$,

(b) Given further that $r_1 = 6$,

(i) find the perimeter of the shaded region
(ii) find the area of the shaded region.

Q6 The diagram shows a sector $OPQRS$ with centre O. Arcs PQ, QR and RS have the same length. Given that OP = 2 m, MN = 0.728 m and

$\angle POS = \frac{2\pi}{3}$,

(a) show that OM = 1.064 m,

(b) find the perimeter of the shaded region,

(c) find the area of the shaded region.