Simple Probability

Probability Trivial

In any one year, the probability of being hit by a meteorite is 5.88×10^{-10} *or* $\dfrac{1}{1,700,000,000}$.

The probability of you getting struck by lightning is greater than that of you striking a lottery.

Can you help Peter?
Peter is given 21 red roses and 7 blue roses to be distributed among three boxes with at least one rose in each box. Jane, blindfolded, randomly selects one of the boxes and then a rose from within the box selected. Peter will be granted a date only if the selected rose is a red one. *How should Peter distribute the roses to maximize his chances of getting a date?*

How do Mathematicians quantify Probability?

In probability, any observation or measurement of a random phenomenon is an ***experiment***.
The possible results of the experiment are called ***outcomes*** and the collection of all possible outcomes is called the ***sample space***.

The probability of an event $= \dfrac{number\ of\ outcomes\ of\ the\ event}{total\ number\ of\ possible\ outcomes}$

Example 1

A die numbered from 2, 4, 6, 8, 10 and 12 is tossed. Find the probability of getting a “4”.

Sample space = {2, 4, 6, 8, 10, 12} implies total number of possible outcomes is 6.

$P(getting\ 4) = \dfrac{1}{6}$

Example 2

A coin is tossed, find the probability that it will land tails up.

Sample space = {Head, Tail} implies total possible outcomes is 2. $P(tails\ up) = \dfrac{1}{2}$

What are the Properties of Probability?

Let $P(E)$ denotes the probability of an event E happening hence $P(E) = \dfrac{n(E)}{n(S)}$, then

(a) $0 \le P(E) \le 1$, in other words it is impossible to get negative probability or probability greater than one.

(b) If E is an impossible event, then $P(E) = 0$.

(c) If E is a certain event then $P(E) = 1$.

Example 3

A die numbered from 1 to 6 is tossed. Find the probability of getting a 9.

$P(getting\ 9) = 0$ (an impossible event)

Example 4

A bag has 3 red marbles. A marble is randomly selected, what is the probability that a red ball is selected.

$P(red\ ball\ selected) = 1$ (a certain event)

Three Rules of Probability

Rule 1 : Addition Rule of Probability (for mutually exclusive events)

Two events are called *mutually exclusive* if the occurrence of one precludes the other event from occurring. In a Venn diagram, the two sets would not intersect and the chance of them happening at the same time is zero. In other words, $P(A\cap B)=0$ i.e. $P(A\cup B)=P(A\ or\ B)=P(A)+P(B)$.

Example 5

A die numbered from 1 to 6 is tossed. Find the probability of getting either 2, 5 or 6.

$$P(2\cup 5\cup 6)=P(2)+P(5)+P(6)=\frac{1}{6}+\frac{1}{6}+\frac{1}{6}=\frac{1}{2}$$

Rule 2 : Multiplication Rule of Probability (for independent events)

Two events are said to be independent if one event has no effect on the probability of the other.

If *A* and *B* are independent events, then $P(A\ and\ B)=P(A\cap B)=P(A)\times P(B)$

Example 6

A bag contains two red marbles and three white marbles. A marble is chosen at random and then put back into the bag. If the process is **repeated twice**, find the probability that

(i) a red marble was chosen every time,

$$P(Red\cap Red)=P(Red)\times P(Red)=\left(\frac{2}{5}\right)\times\left(\frac{2}{5}\right)=\frac{4}{25}.$$

(ii) a red marble was chosen followed by a white ball,

$$P(Red, then\ white)=P(Red)\times P(white)=\left(\frac{2}{5}\right)\times\left(\frac{3}{5}\right)=\frac{6}{25}$$

(iii) each colour was chosen.

P(each colour chosen) = P(1st red, 2nd white) or P(1st white, 2nd red) – mutually exclusive

= P(1st red, 2nd white) + P(1st white, 2nd red)

$$=\left(\frac{2}{5}\right)\times\left(\frac{3}{5}\right)+\left(\frac{3}{5}\right)\times\left(\frac{2}{5}\right)=\frac{12}{25}$$

Example 7

A bag contains two red marble and three white marble. If two marbles are chosen from the bag, find the probability of

(i) getting two red marbles, **(ii)** getting one red and one white marbles.

***Solution* : (i)** $\left(\frac{2}{5}\right)\times\left(\frac{1}{4}\right)=\frac{1}{10}$,

(ii) P(*one red and one white*) = P(1st red, 2nd white) or P(1st white, 2nd red) = $\left(\frac{2}{5}\right)\times\left(\frac{3}{4}\right)+\left(\frac{3}{5}\right)\times\left(\frac{2}{4}\right)=\frac{3}{5}$

Rule 3 : Complementary Rule

If the probability that an event will occur is $P(E)$ and the probability that it will not occur is denoted by $P(E')$ where E' is the complementary event of E then $P(E')=1-P(E)$

Example 8

A die numbered from 1 to 6 is tossed. Find the probability of not getting a 4.

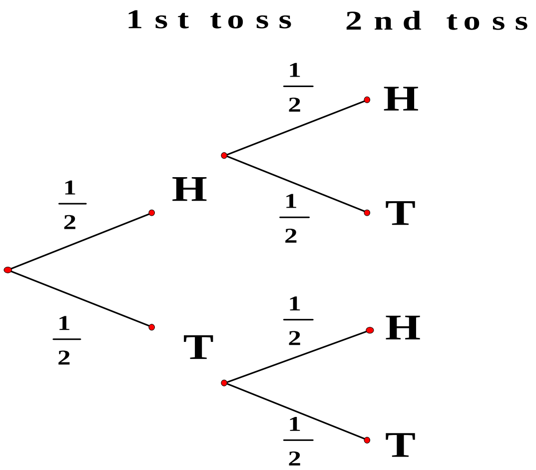
$$P(not\ getting\ 4)=1-P(getting\ 4)=1-\frac{1}{6}=\frac{5}{6}.$$

Note : This is a very useful rule to bear in mind when to compute $P(E')$ directly is very tedious.

Some useful tools for Probability

(I) Probability Tree Diagram

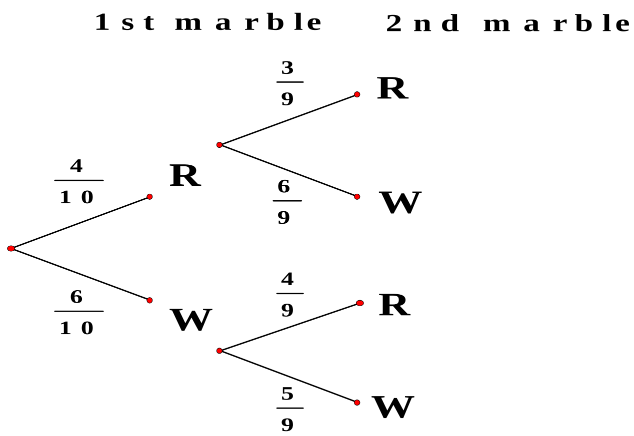
In a tree diagram, the ***probability*** is written on the ***branches*** while the ***outcomes*** are written ***at the ends*** of the branches.
A coin is tossed twice. The tree diagram below shows the possible outcomes and their probabilities.



Example 9

A bag has 4 red marbles and 6 white marbles. Peter takes two marbles at random from the bag, one after the other, and the marbles taken are **not replaced**.
(i) Draw a tree diagram to show the possible outcomes and their probabilities.
(ii) Find the probability that
 (a) the two marbles selected are of the same colour,
 (b) one marble will be red and the other white.

(i)



(ii) (a) P(*both same colour*) = P(*both red*) + P(*both white*) $= \left(\frac{4}{10}\right)\left(\frac{3}{9}\right)+\left(\frac{6}{10}\right)\left(\frac{5}{9}\right)=\frac{7}{15}$

(b) P(*both different colour*) = P(*red*, *white*) + P(white, *red*) $= \left(\frac{4}{10}\right)\left(\frac{6}{9}\right)+\left(\frac{6}{10}\right)\left(\frac{4}{9}\right)=\frac{8}{15}$

(II) Possibility Diagram

Example 10

A tetrahedral die A are marked 2, 4, 6 and 8 on its four faces.
A tetrahedral die B are marked 1, 3, 5 and 7 on its four faces.
The two dice are thrown together and their scores added. Find the probability that the total score is a prime number.

Solution:
The question could be solved easily if we could present all the possible total score systematically in the form of a possibility diagram.

+	2	4	6	8
1	3	5	7	9
3	5	7	9	11
5	7	9	11	13
7	9	11	13	15

From the possibility diagram, it is obvious that 11 out of 16 total scores are prime numbers.

$\text{P}(\textit{total is a prime}) = \dfrac{11}{16}$